4. The Magnetic Field

Macroscopic magnetic effects were already known in ancient times: Magnetite attracts iron particles. This force effect can be described by a vector field in which a defined field intensity is assigned to every point in space, characterized by its strength and direction. Every magnet produces a magnetic field in its vicinity which decreases rapidly with distance. Energy conservation is of course valid: No energy needs to be added for retention of the field (!). If there is a displacement of a piece of iron by the magnetic force, mechanic energy is "gained"; at the same time the magnetic field is weakened. If, conversely, the piece of iron is detached from the magnet, the same amount of energy has to be added which will increase the energy of the field accordingly.

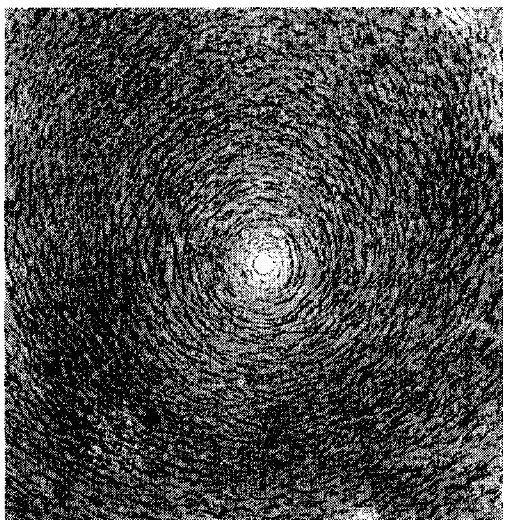
Materials which generate and sustain a permanent magnetic field are called **Permanent Magnets**. This characteristic is predominant for Magnetite (Fe_3O_4) and some other metals. The root cause of the magnetic field is electrons moving around the atomic nucleus and their own intrinsic spin. According to the Bohr-Rutherford atomic model, electrons move in stationary orbits without energy dissipation but produce a magnetic field. A more or less intense magnetic field effect evolves in macroscopic space according to the direction and strength of these fields and the coherence effects of neighboring atoms.

In the same way an electric current flowing through a wire will produce a magnetic field. This field will further increase if the wire is wound to form a coil. However, contrary to the permanent magnet, the electromagnetic field disappears if the current is switched off. The name of this kind of magnet is derived from its operational principle: **Electromagnet**. Permanent magnets and electromagnets have the same effect. Both produce magnetic fields and forces on iron and similar metals. Energetically there seems to be a difference. A permanent current flow is necessary in order to sustain the magnetic field for the electromagnet, which means that energy needs to be supplied. However, one has to distinguish between the one-off portion of energy which is needed to build up the field and the continuous supply of energy which heats up the wire (current × voltage × time = energy). In an ideal conductor (superconductor), the magnetic field could be sustained permanently without the continuous addition of energy.

In addition to the force effect of magnetic fields, there is also the effect of **Magnetic Induction**. A change of the magnetic field over time produces (induces) a voltage in a wire coil. This effect is exploited in a magnetic **Guitar Pickup,** in which a vibrating steel string changes the magnetic field of a permanent magnet, inducing a voltage in the coil of the pickup. Knowledge from several areas is helpful to understand the principles of the pickup, in particular *Magnetostatics,* which describe the stationary magnetic circuit (magnetization of the string), *Magnetodynamics,* which describe time-variant changes in magnetic fields (induction effects), and the *Two-port* and *Systems Theories* which are needed to describe the transfer behavior as a function of frequency. The following chapters will introduce these three disciplines in detail.

4.1 The Basics of Magnetostatics

We will start the following considerations with an electromagnet because the causal relation between field-generating current and resulting magnetic field are clearly visible. Electromagnets do not play any role for pickups, but the results that are obtained can be easily transferred to the permanent magnets which are used in pickups.



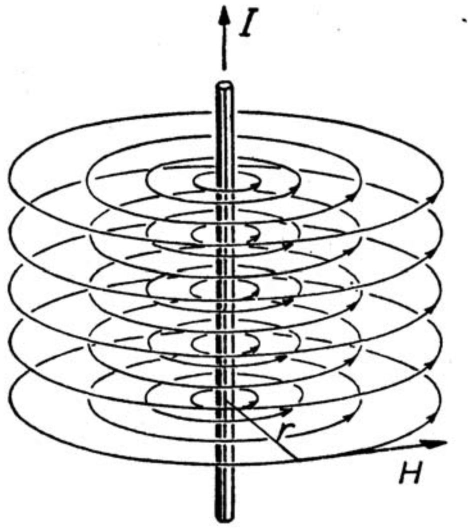


Fig. 4.1: The magnetic field surrounding a current carrying wire; iron filings (left), field lines (right); [18, 19].

When an electric direct current flows through a very long, straight wire, a circular magnetic field is generated around it. The effect of the magnetic field can be visualized by elongated iron filings which are introduced into the space surrounding the wire. The filings line up into circular lines, concentrically wrapped around the wire (**Fig. 4.1**). In this visualization method, the circular lines are not perfectly aligned but easily recognizable by eye. Using the iron filings, a method to visualize the invisible magnetic field had been discovered. The lines marked by the iron filings (circular in this example) were designated **field lines**. The magnetic field does, of course, not only exist within the field lines, rather it fills the entire space. The line-like description is a discrete visualization of a (continuous) vector quantity equally distributed in space.

The evolution of circular structures has two origins. The elongated filings are oriented in a tangential direction by the magnetic field (normal to the position vector) and they arrange themselves into groups connected together at their ends. Iron filings are a good medium to visualize the effects of the otherwise invisible magnetic field. However, an exact quantitative description of the field is not possible with this method. Nevertheless, the empirically deduced circular form is the basis of an abstract analytical description of the field, called the **magnetic field strength** $\vec{H}$. The word "magnetic" is sometimes omitted if no confusion with the electrical field strength is possible.

In the example of a long wire with current flow in one direction, the vector of field strength points in the direction of the field lines, tangential to the circles or normal to the position vector. The value of the field strength vector decreases proportional to 1/r with increasing distance. However, before we start with the exact calculation we must first define the reference systems.

The magnetic field is a **vector field** and the descriptive field-parameter $\vec{H}$ has a value and a direction. Not every field has a vector character. For example, a spatial temperature distribution is described by a **scalar field** with every point having a value but no direction. The **direction** of a vector is given by an angle deviation with respect to a **reference system**. In a two-dimensional scheme **polar coordinates** are particularly suited for the description of the direction. The direction of a vector originating from zero is defined relative to the abscissa, with angle deviations being counted positive in the counter-clockwise (CCW) direction. The spherical coordinates are defined in a similar way in three dimensions. The definition of the positive CCW direction fits into other coordinate systems (Cartesian, complex e-function and Euler) but, ultimately, it is arbitrary: coordinates based on the clockwise direction would also be possible. However, once the **sense of direction** is defined, it has to be maintained throughout the following considerations. The direction of a magnetic field, i.e. the direction of the magnetic field vector, is defined by the tangent to the magnetic field line at every point in space. A tangent, however, is a straight line and not a ray. Consequently, there are two possible reference directions 180° to each other.

The directional reference system for magnetic fields valid today has an historical foundation. It is derived from the needle of a compass. The **Earth** is a huge permanent magnet, producing a weak magnetic field between the North and South Poles. If a compass needle (a little bar-shaped permanent magnet) is suspended so that it can move without restriction, the magnetic force will turn it to be parallel to the field lines. The part of the compass needle that points to the geographic North Pole was defined as magnetic north pole of the compass needle. At the same time it was deliberately defined that the field lines emerge from this **Magnetic North Pole**. This definition, however, yields that the geographic North Pole[§] must be a magnetic South Pole! In the following the North Pole is always the Magnetic North Pole. As for the relationship between current and magnetic field direction we also have to define direction conventions. In metallic conductors the term current flow designates the flow of free electrons (electrical current = charge displacement over time). The direction <u>opposite</u> to the electron flow is called the **technical current direction** (plus to minus within the electrical load). In graphical representations this technical current flow direction is often depicted by an **arrow**. The relationship between the above mentioned current and magnetic field direction can readily be visualized with the **right hand rule**: if the thumb points into the direction of the current flow the other (fisted) fingers will point in the direction of the circular magnetic field.

The field lines of an infinitely long straight conductor are concentric circles centered on the axis of the conductor. This field is called **parallel-plane**, because the same circular field line schemes will evolve on all planes which are in parallel to each other. The computation of this simple scheme is easy but has one disadvantage in that it does not exist in reality because an infinitely long conductor does not exist. Real magnetic fields can have considerably more complicated structures, which can usually be described by, mostly rough, approximations. Finite element modeling (FEM) programs, that divide the fields into small sections, may provide solutions, but will soon reach their limits in the case of fields relevant in, and around, pickups. In the following chapters we will first describe the basic relationships in an idealized manner. The particularities of pickups will be addressed at the end.

[§] Between the Geographic North Pole and the Magnetic South Pole there is a distance of about 1400 km. In central Europe the magnetic field lines have an inclination angle of approx. 65° with reference to the surface. The magnetic flux density is approx. 45µT.

The magnetic field originating from a single wire carrying a current is relatively weak. A strong magnetic field emerges, by superposition (addition) of the individual fields, only if the wire is wound to a coil. The superposition principle is depicted in **Fig. 4.2**. In this case, we have two parallel wires with an equal *amount* of current flowing in *directions* that are opposite to each other. Usually the *technical* current flow is defined from positive to negative. In the cross section, a current flow into the picture plane is represented by a cross ⊗, and the opposite direction out of the picture plane towards the viewer's position is marked by a circle with a point ⊙.

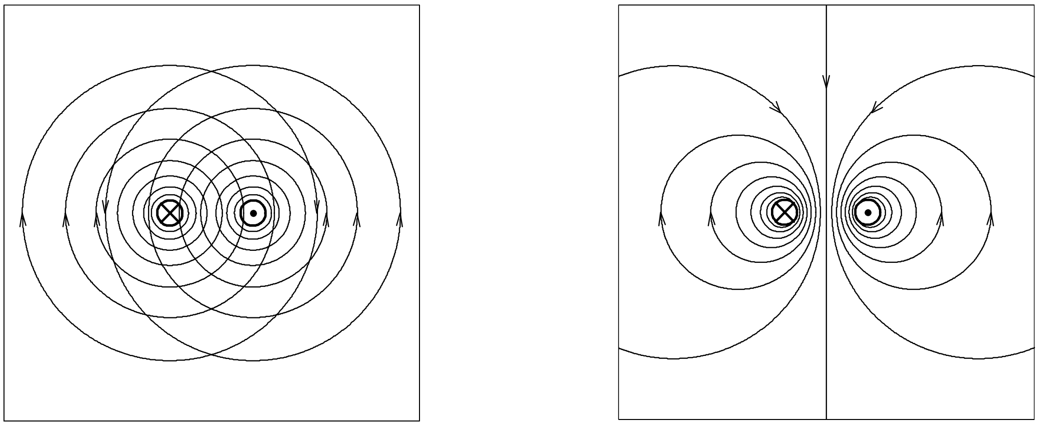


Fig. 4.2: Magnetic field of two parallel wires; anti-parallel current direction. Individual fields (left), superposition (right)

Every wire produces a circular magnetic field propagating with the speed of light. The delays related to the propagation speed are virtually negligible for the small dimensions of a pickup (< 10 cm) and the low frequencies (< 20 kHz), Thus, a quasi-stationary magnetic field can be assumed and no magnetic wave equations are necessary. The magnetic fields of both wires have to be vector-added at every point in space resulting in the eccentrically circular lines. Instead of the "superposition of magnetic fields," we can also speak of the "vector summation" of the magnetic field strength originating from both wires. However, this superposition is only valid for a **linear system**. Air permeated by a magnetic field has linear characteristics, iron has not. First, we will address the linear systems.

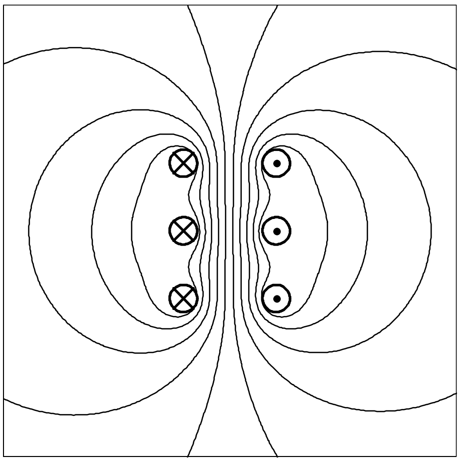
The magnetic field around a current carrying conductor has some characteristics which can be immediately and intuitively understood. It is proportional to the current, decreases with increasing distance and has rotational symmetry with respect to the conductor. Formally the scalar value of the vector of the magnetic field at the point of measurement can be determined by

$$H = \frac{I}{2\pi r},$$

The magnetic field strength outside a straight conductor

in which H is the scalar value of the field strength, I is the current strength and r is the shortest distance of the measuring point to the conductor axis. The formula is only valid for the space outside an infinitely long straight conductor. Again, it should be stressed that, in the case of two wires (Fig. 4.2), the scalar values cannot simply be added. Rather the field strength has to be a *vector* sum. If, for example, two equally large field vectors are normal to each other, the scalar value of the total field strength is not doubled, but only increases by a factor of $\sqrt{2}$.

The scalar value of the magnetic field strength can be increased if the current is increased or if several wires are acting together. **Figure 4.3** depicts several parallel current carrying wires. It is clearly visible that the field lines in between the wires are focused into a channel. A similar, but not identical picture can be obtained, if *one single* wire is wound up into three screw-like coils.



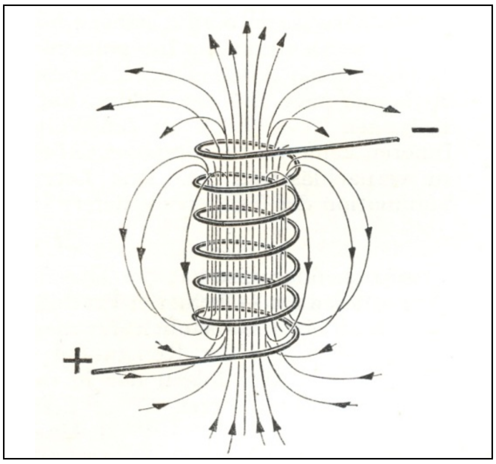


Fig. 4.3: Cross section through the spatial magnetic field of 6 current-carrying parallel wires (left). The spatial magnetic field of a current carrying coil (right) [19].

In Fig. 4.2 and 4.3 the field lines are used as visualization of the invisible magnetic field. The tangent to the 3-dimensional **field line** marks the *direction* of the field strength and the distance between the drawn field lines marks the *magnitude* of the field strength. The shorter the distance between neighboring field lines, the higher the magnetic field strength. The scaling factor can be chosen deliberately: Whether, the lines are drawn, e.g. with a distance of 1 mm or 5 mm for a magnetic field strength of 500 A/m, only depends on the clarity of the description of the total field distribution. The real magnetic field is of course not restricted to the drawn field lines but is continuously distributed in space.

Thus, the field lines do not represent points of equal field strength, so they should not be confused with the isobars of a weather chart or the lines of a contour map. Rather, a curve becomes a field line because the vector of the field strength $\vec{H}$ is a tangent vector on every point of the curve. The *direction* of the field is defined for every point in space by the differential quotient of the vector of field strength. From a geometrical point-of-view, the integration of this spatial differential equation represents the connection of differentially small direction arrows into integral curves, i.e. field lines.

Field lines are curves of equal field strength only in very simple cases such as in Fig 4.1. In general, the value of the field strength changes when moving along a field line. Thus, it stands to reason to examine the line integral over $\vec{H}$ because the field strength is a line-specific quantity. Calculation of the line integral means following a field line and integrating the product of the field strength and the differential (small) line-length *ds*. The field strength is the tangent vector to the field line along the line and, therefore, $\vec{H}$ is always parallel to *ds*. The quantity which is calculated by the line integral is called the **magnetomotive force** $\boldsymbol{V}$ in analogy to the case of the electric field. If one chooses to evaluate the line integral not along the field line, but on a general space curve, one has to calculate the scalar product of $\vec{H}$ and $d\vec{s}$.

In contrast to electric field lines, magnetic field lines do not have an origin and an end. In most cases they form closed loops, but infinitely long complex space curves are also possible. The integral along a closed loop field line, called the **contour integral,** yields the **magnetomotive force**. This force corresponds to the electric current confined by the field line, in other words the source of the magnetic field. This relationship can be easily seen in Fig. 4.1: In an infinitely long wire in which a current I is flowing the field strength at a distance r from the wire is $H = I / (2\pi r)$ and the contour integral along the circumference of a circle with radius r yields I.

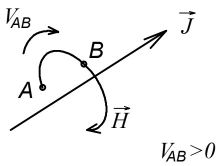
Even in the case that the contour integral does not run along a field line, but along an arbitrary *closed* path in space, its value represents the enclosed current. In this case the scalar product has to be applied, since the field strength vector is not necessarily pointing in the direction of the closed loop. The current passing through the area defined by the contour path is given by the surface integral of the **current density** $\vec{J}$. This surface integral is called the **magnetic flux** Θ. With it, it is possible to establish a relationship between the electrical origin $\vec{J}$ of the field and the magnetic effect $\vec{H}$:

$$\Theta = \int_S \vec{J} \cdot d\vec{S} = \oint_s \vec{H} \cdot d\vec{s}$$

Magnetic flux law (Laplace Law)

In this equation $\vec{J}$ is the vector of current density (Amperes/Square Meter)[§], $\vec{H}$ is the vector of magnetic field strength (Amperes/Meter). The flux passes through a surface S defined by the contour line s. $d\vec{s}$ is an infinitely small linear element of this contour line; $d\vec{S}$ is an infinitely small surface element of the entire surface delimited by s. The surface element is defined as a vector: The scalar *magnitude* of this vector is the surface area; its *direction* is perpendicular to the surface element. The product of the vectors is the **scalar product**, which is defined as the product of the vector *magnitudes* multiplied by the cosine of the angle between the vectors. The circle on the integral symbol indicates that integration has to be carried out along the *closed* curve s, i.e. a *contour integration* needs to be applied. If the contour integral does not run along the entire (closed) circumference, but rather along a curve between two points A and B, one obtains the **magnetomotive force** V:

$$V_{AB} = \int_A^B \vec{H} \cdot d\vec{s}$$



Magnetomotive force

The magnetomotive force is derived from a scalar product and is, consequently, a scalar. Scalars do not have a direction but they do have an orientation (also called direction character). $V_{AB} = -V_{BA}$ is valid. Most often the orientation is depicted by an arrow: The sign of the magnetomotive force is positive if the potential (4.2) decreases with the direction of the arrow; in this case the direction is identical with the magnetic-field-strength direction. If one points with the thumb of the right hand into the $\vec{J}$ - direction (technical current direction), then the bent fingers point into the V-direction.

[§] Sometimes J is also used for the polarization or the magnetic dipole moment – this is not meant here. In addition the current density is sometimes called j; j is used for $\sqrt{-1}$ here.

4.2 Magnetic Potentials

The magnetic field strength, as defined in chapter 4.1, is a differential length-specific quantity whose line integral yields the magnetomotive force. This figure can be interpreted as an integrated quantity (along the line) as well as the difference between the **scalar potentials** associated with the start and end points of the line. The potential defines the "magnetic power" of every point in space, whereas the magnetic field strength describes the spatial change of this "power". The word potential is derived from the Latin word "potentia" which means "ability, force, power, influence". The definition of a potential is also common in other areas, e.g. the gravitational field can be derived from the "potential energy". However, assigning an absolute power to every point in space within the framework of a relative scale immediately leads to the question about the **zero point** of this scale. In the case of temperature, there is an absolute zero deduced from energetic considerations. However, for the magnetic field this scaling is arbitrary. Strictly speaking the magnetic potential is not defined by a relative scale but rather by an **interval scale**, with zero being defined by a constant deliberately chosen for convenience. If one computes the magnetic field or the magnetomotive force as a potential *difference*, this constant will disappear. This leads to the legitimate question why a pseudo-absolute quantity (potential) is defined, if one continues to work with differences (intervals). The explanation can be found less in the area of physics but more in mathematics. The field and potential theory, which is based on complex function theory, offers a universal tool for the description of all fields, independent of their individual scaling.

In the **scalar potential** and the **vector potential**, mathematics provides us with two abstract quantities whose physical interpretation is somewhat arduous. First of all an obvious misinterpretation has to be ruled out: Even though the magnetic field is a vector field, it has both a vector potential <u>and</u> a scalar potential. The vector potential is a vector quantity associated to every point in space, the scalar product is a scalar quantity associated to every point in space. The scalar potential is, however, not the scalar value of the vector potential.

The **scalar potential** ψ is the quantity which leads to the magnetomotive force V through the formation of differences. If the distance between two points approaches zero, the respective potential difference converges towards the magnetic field strength $\vec{H}$. Hence, the differential quotient to be determined is the **gradient**:

$$\vec{H} = -\operatorname{grad}\psi = -\begin{pmatrix} \partial\psi/\partial x \\ \partial\psi/\partial y \\ \partial\psi/\partial z \end{pmatrix}$$

Magnetomotive force as a function of the scalar potential. The unit of the scalar potential is the Ampere.

The scalar potential ψ is, as suggested by its name, a scalar, the gradient is a vector. It points along the direction of the highest field *growth*. The field strength vector $\vec{H}$ points along the direction of the highest field *decrease* (H-decrease) since the equation contains a minus sign.

The gradient of a constant is zero. As the gradient formation is a linear operation, an offset does not change the gradient. As a consequence, the (arbitrary) definition of the potential zero has no influence on the field strength: grad(ψ) = grad(ψ + const).

It is easy to deduce the field strength from the scalar potential by calculating the gradient (spatial differentiation). Conversely, one has to calculate the line integral in order to deduce the scalar potential from the field strength. As always, integration needs an additive constant – the latter defines the absolute potential-zero. In the following equation this potential-zero is assigned to the point in space P_0. A line integral has to be formed between P and P_0:

$$\psi(P) = -\int_{P_0}^{P} \vec{H} \cdot d\vec{s} = \int_{P}^{P_0} \vec{H} \cdot d\vec{s}$$

Scalar potential as function of the magnetic field strength. $\psi(P)$ is the scalar potential at point P. At point P_0 the scalar potential is arbitrarily set to zero.

The **magnetic** scalar potential exhibits a specific feature: It is not **defined** universally and, where it is defined, it is either discontinuous or **ambiguous**. No scalar potential is allowed in sections of space where an electric current density $\vec{J} \neq 0$ is present. Inside an electric conductor no scalar potential exists. Not that it is zero, rather it is not defined. Outside the conductor a scalar potential can be defined, e.g. in the air, which is considered to be an insulator. If one defines the potential reference point [$\psi(P_0) = 0$] at a point P_0 outside of a straight current-carrying conductor and circles the conductor on a circular line, the potential will assume positive values. After a full circle one again arrives at P_0. The potential at this point equals the magnetomotive force. After two revolutions (arriving at the same point!) it amounts to twice the magnetomotive force. The scalar potential defined this way is continuous but ambiguous. Alternatively, one could restrict the definition range to one single full revolution. Then the scalar potential would become unique but would be discontinuous, because it changes its value abruptly at the borderline.

The second method is used frequently, i.e. a unique but (spatially) discontinuous scalar potential. For this, a sector or domain is defined in which an electrical current flow is not allowed (here $\vec{J} = 0$ is valid), and boundary lines are introduced so that this area will become **"simply connected"**. In a simply connected area *every* closed path may be reduced to a point. In **Fig. 4.4** the area outside the conductor is such a sector if the border line is introduced as a section boundary. It prevents a multiple circulation around the conductor, but at the same time produces a discontinuity (at the direct transition from C to A).

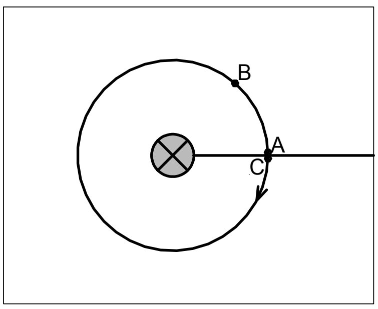


Fig. 4.4: Simply-connected area around a current carrying conductor. The line to the right is a sector boundary. The scalar potential will grow from A over B to C. The arrow indicates the direction of the *H*-vector.

It might be seen as disadvantage that the scalar potential is only defined outside the conductor. However, it does have the advantage that one (univariate) scalar is sufficient to describe of the field instead of the three field strength components (H_x, H_y, H_z) that would be otherwise necessary.

The **magnetic vector potential** is defined in addition to the magnetic scalar potential. It enables field descriptions inside as well as outside the conductor. However, the magnetic vector potential is not a very clear and accessible quantity. In fact, its existence is derived from formal mathematical considerations and subsequent numerical (FEM) calculations of the field (Potential and Field Theory, 4.9). The calculation of two-dimensional fields with the FEM-software "ANSYS" is only feasible with the vector potential and not with the scalar potential. The **vector potential** $\vec{A}$ is dependent on the field strength $\vec{H}$ via a special spatial differentiation, the rotation or curl:

$$\mu \cdot \vec{H} = \nabla \times \vec{A} = \operatorname{rot} \vec{A}$$ Vector potential[§] $\vec{A}$

Here μ is a material constant, the so-called permeability (chapter 4.3). In Cartesian coordinates the **rotation** is calculated as the difference of partial differentials and can be depicted with the nabla operator ∇:

$$\operatorname{rot} \vec{A} = \begin{pmatrix} \partial A_z / \partial y - \partial A_y / \partial z \\ \partial A_x / \partial z - \partial A_z / \partial x \\ \partial A_y / \partial x - \partial A_x / \partial y \end{pmatrix}$$

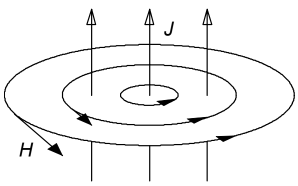
Curl in Cartesian coordinates.
The unit of the vector potential[§] is Vs/m.

For magnetic fields that can be represented by a **two-dimensional** scheme, e.g. parallel-plane fields, the vector potential has only one component. Both of the other components are zero. For example, the H_z-component is zero for an H-field only defined in the xy-plane. In the associated vector potential only A_z is non-zero. This is the component of the potential which is perpendicular to the xy-plane.

Fig. 4.4 represents such a parallel-plane field. The current flows into the plane of projection and an H-field emerges in the xy-plane. The vector potential has only an A_z-component parallel to the current flow. The equation specified simplifies to:

$$\mu \cdot \vec{H} = \nabla \times \vec{A} = \operatorname{rot} \vec{A} = \begin{pmatrix} +\partial A_z / \partial y \\ -\partial A_z / \partial x \end{pmatrix}$$ 2D-vector potential

The vector potential presents an elegant method to define boundary conditions. This is necessary, for example, to reduce the complexity of computations or to set boundaries to infinite domains in FEM calculations. In addition, it is relatively simple to define field lines with the vector potential (chapter 4.7). **Figure 4.5** depicts the spatial vector relationship between the current density and field strength, and the flux density and vector potential, respectively.



$$rot\ \vec{H} = \vec{J}$$

$$rot\ \vec{A} = \vec{B}$$

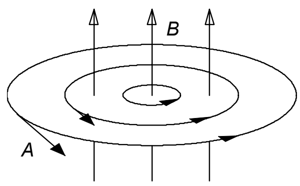


Fig. 4.5: Spatial relationship between $\vec{J}$ and $\vec{H}$ (left) and $\vec{B}$ and $\vec{A}$ (right).

[§] The symbol $\vec{A}$ must not be mixed up with the area vector!

4.3 Matter in Magnetic Fields

It has been found to be useful to describe fields, in analogy to fluxes of matter (water circuit), by **potential** and **flux quantities.** The expressions *flow* and *flowing* are to be used in a figurative sense. There is no real flow in the magnetic circuit, contrary to the water circuit. The scalar pressure is the quantity of drive in the water circuit. If the pressure is not equal in the entire fluid, but varies as a function of position, there are vector pressure differences or gradients and forces acting on the fluid particles which, as a consequence, move or flow in the opposite direction to the gradient. Hence, the pressure can be interpreted as a scalar potential in which its gradient would be comparable with the field strength. The velocity of the fluid, however, cannot be deduced directly within this scheme. Other characteristics of the fluid, like viscosity and inertia as well as the boundary conditions, have to be considered.

An **electric** circuit is quite similar. The gradient of the electric scalar potential is the electrical field strength and its line integral is the electric voltage. The material quantity "impedance," or the admittance, has to be known in order to deduce the current flow from the voltage or the electrical field. This is not different for the **magnetic circuit**. The magnetomotive force or, alternatively, the magnetic field strength, is the quantity of drive and the magnetic resistance determines the amount of magnetic flux. As already mentioned this flux is immaterial and, like all the other magnetic quantities, is not visible. As long as the entire magnetic field is confined in a single material, the introduction of a magnetic flux could be dispensed with. The introduction of the flux quantity is advantageous if several materials have to be considered. The **continuity condition** is especially useful. It means that the entire incoming node flux is zero for an incompressible liquid. If, for instance, a node is formed by three tubes and in the first tube the incoming flux is 5 m^3/s, in the second tube the incoming flux is 4 m^3/s, then, consequently, the incoming flux in the third tube has to be -9m^3/s, i.e. the outgoing flux at the node is +9 m^3/s. This law is also known as Kirchhoff's (nodal) rule or Kirchhoff's first law. The electric current divides itself at a conductor node and the magnetic flux at a material node based on the same principle.

The flux quantity already yields clear descriptions without the presence of nodes. If the cross-section of a tube with impermeable walls (!) varies, or the flow resistance depends on location, there is still the same flux through every cross-section, given that the fluid is incompressible. This is equivalent to electrical engineering: The same current flows through serial resistances even though their ohmic values might be different.

The potential quantity is defined in integral and differential form. The integral quantities in the water circuit are the pressure and the local pressure difference. The differential value is the pressure gradient. The flux quantity is also defined as integral and differential, as the total flux in the water circuit, e.g. m^3/s, and as the **flux density**, i.e. flux per transverse section (m/s). The relationship between the differential potential and the flux quantity is established via the **specific conductivity** or the reciprocal **specific resistance**. In this case "specific" means material specific as well as volume specific.

The quotient of pressure gradient and flux density is the specific flow resistance in the water circuit. If the flow resistance is large, the water flow is low. A higher fluid viscosity leads to a higher specific resistance and a slower current. For the electrical current, the quotient of the electrical field strength (in V/m) and the current density (in A/m^2) yields the specific resistance (in Ωm). Poor conductors have a high specific resistance, i.e. they are "highly resistive". The specific conductance is defined reciprocally to the specific resistance. In an electric circuit it is the quotient of current density and electric field strength.

The specific conductivity in the **magnetic circuit** is called the **permeability** μ, which is the quotient of the magnetic flux density B and magnetic field strength H:

$$B/H = \mu = \mu_r \cdot \mu_0 \qquad \mu_0 = 4\pi \cdot 10^{-7}\,\frac{\mathrm{Vs}}{\mathrm{Am}} = 1.257\,\mu\mathrm{H/m}$$

Permeability μ

In many cases the permeability μ is divided into two factors, the absolute permeability μ_0 and into the dimensionless relative permeability μ_r. The absolute permeability, which is also called the magnetic field constant, has the unit Vs/Am, or Henry / Meter (H/m). Care has to be taken here. The italic H is the equation symbol for the field strength, the non-cursive H stands for the unit Henry (1H = 1Vs/A). Sometimes the unit Henry is also abbreviated by Hy to avoid confusion. µH means one **microhenry**, which is 10^{-6} H. Again one has to differentiate: The italic μ is the quantity of permeability, the upright µ is a prefix which means "one millionth".

The relative magnetic permeability of the vacuum is 1. Thus μ_0 can be interpreted as the permeability of the vacuum. The absolute permeability μ_0 can also be applied to air with a high accuracy. For many materials the relative permeability μ_r shows only a minor deviation from 1. These are called non-magnetic materials. In physics one further distinguishes between paramagnetic and diamagnetic materials but this discrimination is not necessary here. For **magnetic materials** (magnet materials) $\mu_r >> 1$ is valid. This holds for all iron and steel parts and the permanent magnets of an electric guitar. Magnetic materials that can be magnetized by weak magnetic fields are called **magnetically soft**. The opposite expression is **magnetically hard**. The limit at which a material becomes magnetically hard can only be described approximately (H_C > 1kA/m, see later).

The **permeability** μ is the magnetic conductivity. A material with large μ has a high magnetic conductivity and the magnetic flux density B can become very high even at low field strength. In an electric circuit one would talk about a highly conductive, low resistance material. If materials with different magnetic conduction are located next to each other in the flow direction (parallel), the material with higher conductivity will carry the larger part of the flux. In two parallel resistors the one with the lower resistance will carry the higher electrical current, and if two parallel layers of iron and air are considered, almost the entire magnetic flux will be focused in the iron, because its μ is considerably higher than 1.

The electrical current passing through a transverse area S is $J \cdot S$, or electrical current density multiplied by the area. Likewise, the magnetic flux is the product of magnetic **flux** density and the area. A scalar product has to be formed if the area is not located transverse to the flux density. If the flux density depends on the location, one has to integrate:

$$\Phi = \int_S \vec{B} \cdot d\vec{S} \qquad \vec{B} = \frac{d\Phi}{dS} \cdot \vec{e}_\Phi$$

Magnetic flux Φ

The flux density is the quotient of the flux and of the area it flows through. The flux density vector points into the direction of the Φ-unit vector if the area tends to zero.

The permeability μ is a scalar constant only in very simple cases. μ shows a strong nonlinear dependence on H in most cases for which μ deviates considerably from 1. Very large values of μ can be obtained (above 10000) for small values of H. The material will become "magnetically saturated" with increasing field strength and μ_r decreases. Consequently, the magnetic field can no longer be considered as a linear system, which has far-reaching consequences: **non-linear distortions** emerge, the superposition principle is no longer valid and there is no transfer-function and impulse response. In addition, time invariance can no longer be assumed because the memory of permanent magnetic materials yields a hysteresis: for increasing field strength the flux is different from the decreasing case. Finally, one has to consider that, at least in strong magnets, the permeability becomes orientation-dependent; μ_r will become a tensor in these anisotropic materials.

The material is isotropic and linear for the simplest case. Then μ_r is a constant and the field directions of the $\vec{B}$ - and $\vec{H}$ - vectors are the same: $\vec{B} = \mu \cdot \vec{H}$. However, an approximately linear description is also possible for a non-linear B/H relation (linearization, tangent approximation, Taylor series) for small deviations from linearity. If the amplitude of the signal can no longer be considered as small, an **isotropic/non-linear** model has to be used; in that case μ is defined as an H-dependant series of curves.

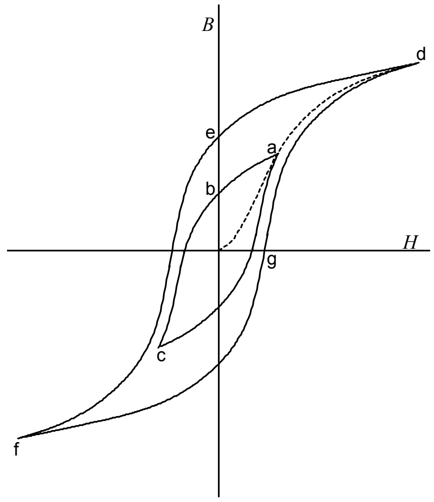
For the **anisotropic/linear** model, μ is indeed independent of H, but depends on the spatial orientation (relative to the crystal axes).

$$\vec{B} = \begin{pmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{pmatrix} \cdot \vec{H} \qquad \rightarrow \qquad \vec{B} = \begin{pmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{pmatrix} \cdot \vec{H}$$

The μ-tensor can be simplified by choosing a suitable coordinate system, so that only 3 elements remain. This can be achieved by orienting the coordinate system along the main axes of the material (which is the direction of the largest μ); the other two corresponding μ-values are then smaller and often equal.

Anisotropic/non-linear materials can only be described with enormous effort. For the simple case, every one of the three μ-components is depicted as H-dependent curve or series of curves. However, this case does not include the existence of non-linear couplings between the three spatial directions. An exact modeling most often fails due to imprecise measurements and a too large a diversity of parameters.

Materials with a large μ_r are called **ferromagnetic** because, in most cases, iron (Ferrum) is the root cause for the magnetisability. Cobalt and Nickel as well as some rare earths and special alloys also show magnetic behavior. A single crystal of iron will show anisotropic behavior. Its μ_r yields the largest values in the direction of the cube edge. However, since all magnetic domains are pointing into different directions in the unmagnetised (virgin) state of iron, the macroscopic magnetic field can be considered as isotropic (quasi-isotropy). An anisotropic behavior can be grown using particular production procedures, e.g. cool down within a magnetic field or crystallization on a quenching plate.



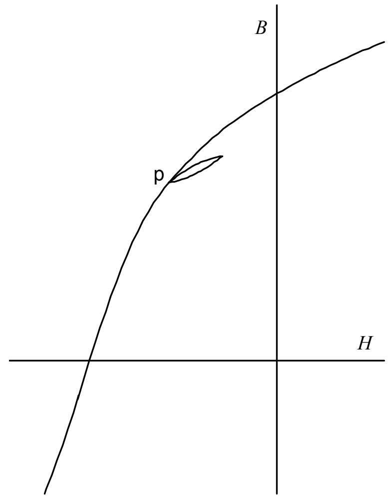


Fig. 4.6: Ferromagnetic demagnetization curves (left).In the demagnetized state, the field strength H and the flux density B are zero (at the origin). Increasing H e.g. towards the point **a**, B will increase according to the dotted initial magnetization curve. However, if H is set to zero again, B will not return to zero but rather to the value at point **b**. Applying a negative magnetic field, one will reach e.g. point **c**, and by reversing the field again point **a**. Further increasing the field, one will reach point **d** via the initial magnetization curve. If now H is set to zero a remanent flux density at point **e** will remain. The picture on the right shows reversible changes at very small amplitudes (reversible permeability).

In **Fig. 4.6** the nonlinear relationship between B and H for a ferromagnetic material is depicted. This so called "**hysteresis**" is not only curved, it also splits into two sections: approaching a certain value of the field strength by increasing H (from the left) will result in a smaller B-value than by decreasing the H-value (from the right). The loop in Fig. 4.6 can only be run through counter-clockwise.

Both the increasing and the decreasing section of the curve converge against a common asymptote for high absolute values of the field strength – the material is magnetically saturated. If the field strength is set to zero from one of these saturation points then a permanent flux density remains at the crossing point with the ordinate axis. This is called the remanent flux density or **remanence**. In Fig. 4.6 the remanence point is depicted by **e**. In order to reduce the flux density to zero a counter field strength must be applied, which is called the **coercivity** or **coercive field strength**. In older literature it is sometimes named the "coercive force". In Fig. 4.6 the coercivity point is the abscissa section of the outermost hysteresis curve and is marked with **g**.

The flux density follows the curves in Fig. 4.6 only if H changes monotonically. If H is decreased from positive values to the point **p,** as shown in **Fig 4.6** (right), and is successively increased again by a small amount, the return run will not take place on the large section of the drawn hysteresis but rather on the lower part of the slanting branch. The return to **p** will be realized on the upper part of the branch. For very small changes around the working point **p** the branch sections will approximately coincide and their slope will yield the **reversible permeability**. It is not given by the differential quotient of the B/H curve, but is smaller (see magneto dynamics).

4.3.1 Magnetically Soft Materials

Magnetically soft materials are characterized by a slim hysteresis, i.e. a small coercive field strength. It is easy to permanently magnetize them, but small external magnetic fields may also change their magnetization to new values. The characterization “magnetically *soft*” is chosen as to depict this easy (magnetic) access and does not necessarily mean reduced mechanical hardness. **Iron** is the most common soft magnetic material. The crystal structure is also responsible for the magnetic characteristics in addition to the chemical constituents: cold work hardening as well as soft annealing will change the magnetic properties. Even small amounts of additives will change the mechanical as well as the magnetic ‘hardness’.

The **coercive** field strength of magnetically soft materials is typically below 1 kA/m, in special cases below 1 A/m. The **remanent** flux densities most often lie between 0.8 T and 1.5 T. In special cases they can be below 0.1 T. No single value can be given for the **permeability** because it is strongly dependent on amplitude. The relative permeability of cast iron is in the range of 50 to 500. Special metals may reach over 300,000.

Magnetically soft materials are used in pickups to guide the magnetic flux. The flux originating from a permanent magnet is channeled and focused to the strings by magnetically soft **pole pieces**. These pole pieces can be solid metal blocks but also laminated sheet packages or height-adjustable screws. Some pickups (e.g. Fender, old Stratocasters) also may have no pole pieces at all.

4.3.2 Magnetically Hard Materials

Magnetically hard materials should retain their magnetic field after magnetisation as long as possible without external influence; they need a high coercive field strength. They are also called permanent magnetic materials because their field will last for decades if handled correctly. The **coercive** field strength of simple steel magnets is approximately 5 kA/m, for the Alnico-alloys often used in pickups it is around 32 – 62 kA/m and up to 2000 A/m can be reached with special magnets. The **remanence** is between 0.5 T and 1.5 T. The permeability is, like in magnetically soft materials, strongly dependent on the working point Typical μ_r values are from 1 to 5. Magnets with a high coercive field strength tend to have a smaller μ_r.

4.3.3 Non-Magnetic Materials

Only the vacuum is perfectly non-magnetic. μ_r is slightly smaller than 1 for diamagnetic materials, e.g. 0.99998 for Pb, and μ_r is slightly higher for paramagnetic materials, e.g. 1.00002 for Al. Such small effects are completely unimportant for measurements at pickups and also why materials like wood, copper, aluminum, all plastics (PVC, Nylon), varnish, brass, bronze, are considered as non-magnetic (and also non-magnetizable).

4.4 Pickup Magnets

There are several methods to detect the vibrations of a string and to transfer the movement into an electrical current. One of these methods is based on the induction principle: A magnetic field varying with time induces (produces) an electrical voltage in a conductor loop (wire winding). Pickups based on this working principle are called magnetic pickups. The magnetic field is produced by a permanent magnet and is time-dependently modified by the vibrating string.

The magnets of most Fender and Gibson pickups are fabricated from **AlNiCo** alloys and this basic materials is also of special importance for other manufacturers. Permanent magnets are known from ancient times. However, efficient permanent magnets have been only available since the beginning of the 20th century. At first C-steel magnets were in use and improvements were achieved with Cr and Co-steel. In the middle of the 30s the **Mishima-Metal** (13,5% Al, 28,5% Ni, the remainder Fe) was developed in Japan and a little later the **MK-Alloy** (13Al, 25Ni, 4Cu). At the beginning these alloys were still called *steels*. Nowadays, the word steel stands for carbon containing steel and carbon is undesirable as a constituent in AlNi or AlNiCo alloys so today these are called magnet alloys. **Alnico alloys** contain **Al**uminum, **Ni**ckel, **Co**balt, copper, **Ti**tanium and other additives in addition to the main constituent iron. The first alloys were fabricated without cobalt, so they are sometimes called AlNi magnets, but sometimes also AlNiCo-magnets even though they do not contain cobalt.

The history of the AlNiCo magnets begins around 1935, at a time when the first commercial pickups were developed in the USA. Gibson built a magnetic pickup into the Hawaiian Electric, which contains a huge 11 cm long horseshoe magnet made of steel. The developer was **Walter Fuller**, however the pickup was known as the Charlie Christian pickup after the artist who used it for the first time in public. Alnico magnets were first implemented at Gibson in the 40s. At the end of the 40s Walter Fuller launched a new pickup with a bar magnet and considerably smaller dimensions, the **P 90**, which is still in production. Nearly at the same time Leo Fender started the production of the Broadcaster, which was renamed to **Telecaster** shortly afterwards. It was also equipped with AlNiCo magnet pickups, however the magnets were formed as cylinders.

One of the first Alnico alloys produced in the USA was **Alnico 3** (or Alnico III). The Al content is 12%, with 24 – 26% Ni and 0 – 3% Cu added. Co was not yet included. The somewhat stronger **Alnico 2** alloy contains 10% Al, 17 – 19% Ni, 12 – 13% Co and 3 – 6% Cu. The even stronger **Alnico 5** magnets were available at around the beginning of 1940 with 10% Al, 17 – 19% Ni, 12 – 13% Co and 3 – 6% Cu. In the following years a multitude of new magnet materials was introduced which in the case of Alnico were supplemented with numbers and additional letters. Patents and trademarks protect the mixing recipes and trade names, which leads to an ever growing number of designations: Nialco, Ticonal, Alcomax, Hycomax, Hynico, Ugimax, Columax, Coerzit, Oerstit, Gaussit and many others. In the 50s a new type of magnet became available that does not require expensive alloy constituents. Within a short time **ferrite magnets** make it to the top of the magnet market. With the beginning of the 70s a new class of high-performance rare earth magnets is available with a five times higher energy density. The pickup producers, however, soon realize that strong magnets not only increase the volume but also change the sound. This is why, in the course of a return to old values, it was necessary to declare Alnico as the favorite material again.

4.4.1 Alnico Magnets

Alnico alloys contain 7 – 13% Al, 12 – 18% Ni, up to 40% Co and up to 6% Cu as well as possibly small amounts of Ti, Si, S and Nb. Alnico 5 (or Alnico V) is often mentioned in connection with guitars. This numbering system (Alnico 1 -12), typical for the USA, should classify the increasing BH_{max} value (volume-specific energy), however, a precise specification of the magnetic characteristics and composition is not possible. Particularly, one has to take into account that Alnico 2 is stronger than Alnico 3. Alnico 2, Alnico 3 and Alnico 5 are used most often in pickups.

	B_r / T	H_c / kA/m	BH_{max} / kJ/m^3	Al	Ni	Co	Cu	Ti
Alnico 3	0.65 - 0.75	32 - 45	10 - 11	12	24-26	0	0-3	–
Alnico 2	0.7 - 0.85	34 - 52	12 - 14	10	17-19	12-15	3-6	0.5
Alnico 5	1.1 - 1.3	50 - 62	30 - 50	8	12-15	23-25	0-4	0-0.5

Table: Magnetic characteristics and composition percents of Alnico-magnets; remainder = Fe.

Alnico magnets are differentiated in casted and sintered ones, which can be isotropic or anisotropic, depending on their production method. The production of **cast** magnets consists of melting the metallic constituents and casting the melt in the mold where it solidifies, e.g. sand casting, chill casting, and vacuum precision casting. Untreated casted magnets have a dark greyish-brown color. During **sintering**, the fine-milled constituents are baked under high pressure and high temperature. Sintered magnets are shiny metallic, similar to nickel. Contrary to the cast magnets, sinter magnets have improved mechanical but slightly worse magnetic characteristics. In particular, their remanence is slightly smaller than that of cast magnets. The coercive field strengths are similar. Sinter magnets can only be produced economically with small dimensions and in large quantities. They exhibit fewer pores, shrink holes and cracks than cast magnets and better retain their required composition. Alnico magnets can only be ground due to their very high mechanical **hardness** (Rockwell hardness 45 – 60 HRC). The ground surfaces are shiny metallic.

Isotropic material characteristics are independent of direction. In contrast, anisotropy means that a spatially predominant direction exists in which a certain characteristic, in this case magnetic, is more pronounced (*oriented material*). Cast as well as sintered magnets without special treatment are isotropic.

Magnetic alloys with Al, Ni and Co constituents were, and still are, produced world-wide under different brands. As the first commercially successful pickups were developed and wound in the USA, the American abbreviation Alnico became accepted. **Seth Lover**, the developer of the Gibson "Patent Applied For" humbucker, answered the question whether he *always* used Alnico V magnets with "We also used Alnico II and III, because Alnico V was not always available. The only difference was that Alnico V did not lose its magnetization as easily [13]."

There is something to add from a physical point of view and, obviously, also from a commercial point of view: in 2002 Gibson communicated on their homepage: *"BurstBucker pickups now give guitarists a choice of three replica sounds from Gibson's original "Patent Applied For" pickups – the pickups that give the '59 Les Paul Standard it's legendary sound. ... with unpolished **Alnico II** magnets and no wax potting of the coils, just like the originals".* However, one should keep in mind that Alnico II as well as Alinco V were produced in different variations before wondering about the fact that today's replica pickups are produced out of a material that once was a stopgap. C. Heck [21] maintains four different Alnico II and 8 different Alnico V versions:

	B_r / T	H_c / kA/m	BH_{max} / kJ/m^3	Al	Ni	Co	Cu
Alnico II	0.73	46	12.8	10	17	12.5	6
Alnico II A	0.70	52	13.6	10	18	13	6
Alnico II B	0.75	46	13.6	10	19	13	3
Alnico II H	0.84	48	16.8	10	19	14.5	3

	B_r / T	H_c / kA/m	BH_{max} / kJ/m^3	Al	Ni	Co	Cu
Alnico V A	1.20	58	40	8	15	24	3
Alnico V AB	1.25	55	44	8	14.5	24	3
Alnico VABDG	1.31	56	52	8	14.5	24	3
Alnico VB (V)	1.27	52	44	8	14	24	3
Alnico VBDG	1.33	55	52	8	14	24	3
Alnico V C	1.32	46	44	8	13	24	3
Alnico V E	1.10	56	36	8	14.5	24	3
Alnico V-7	1.28	62	56	8	14	23	3

Table: Magnetic characteristics and percent compositions of Alnico magnets; remainder = Fe.

Obviously, a "typical" Alnico 5 material does not exist. The remanence values given in this table vary by ±10% and the coercive field strength by ±11%. The variation of the respective hysteresis curves is shown by **Fig. 4.7**. The units correspond to the CGSA-system common in the USA: 1Oe = 80 A/m, 10 kG = 1T, 1 MGOe = 8 kJ/m^3. When considering whether Alnico 5 "sounds" better than Alnico 2, one also has to investigate which special Alnico variation is applicable. In addition, it is especially problematic that the magnetic characteristics of a material not only derive from its chemical composition but also from the physical parameters of its production process. In particular, the temperature treatments and external magnetic fields can have lasting (permanent) impact.

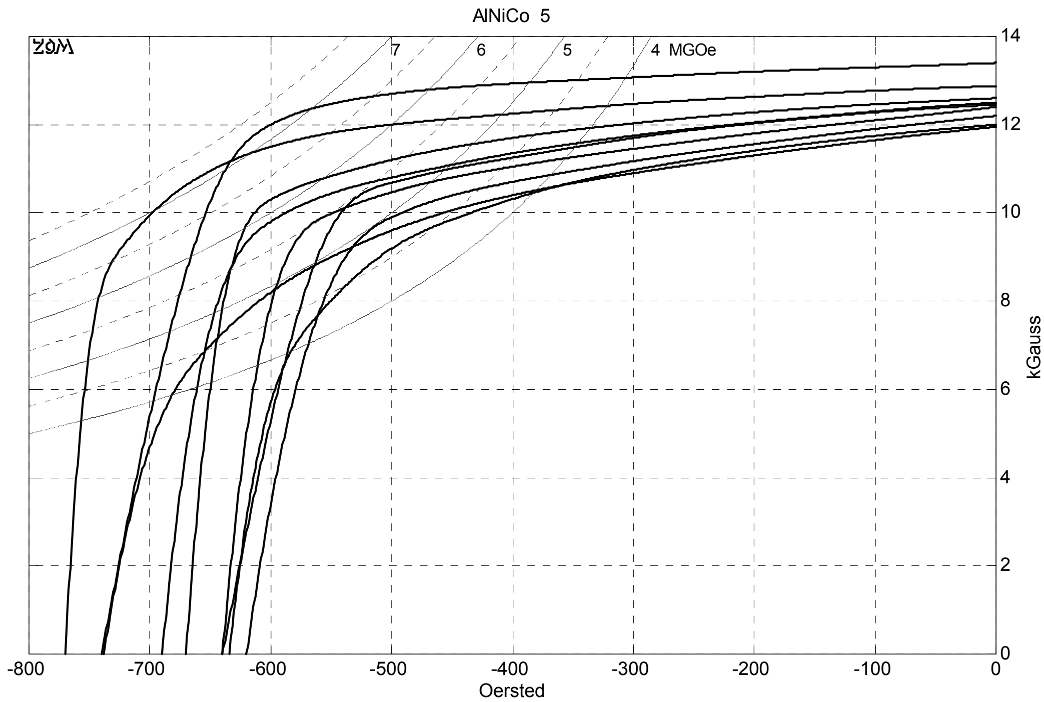


Fig. 4.7: *B/H*-characteristics of various Alnico-5-magnets [22, 23]. 1Oe = 80A/m, 10kG = 1T, 1MGOe = 8kJ/m^3.

The basics of material science are helpful in understanding of the characteristics of Alnico: In solid metals the atoms arrange themselves in a regular periodic lattice. However, this **crystal lattice** is not constructed perfectly, but also contains crystal defects which have a significant influence on the material properties. The supply of energy (heating) results in a rearrangement of the atoms in looser structure and the metal becomes liquid. During the subsequent **cool-down** (solidification), crystallization begins at many different sites (the so-called **nucleation centers**). The growth of these internally regular crystals, also called grains or **crystallites**, persist until they hit a neighboring crystallite. At room temperature the metal has a **polycrystalline structure**. Polycrystalline means that the entire metal volume is made up of many single crystallites that butt up at their grain boundaries. Inside, every crystallite is **monocrystalline**, i.e. all atoms are essentially arranged in a periodic lattice. However, the orientation of each crystallite, which is only several micrometers in size, points into a different direction.

The properties of a crystal lattice result from its constituents, the bonding conditions and the lattice geometry. It is well known, that diamond as well as graphite consist of pure carbon. Both materials, which in fact do not belong to the metals, have completely different characteristics because their carbon atoms are arranged in different crystal configurations (cubic or hexagonal). Likewise, some metals occur in different (polymorphic) crystalline structures: iron, cobalt, manganese, titanium, tin and zirconium. At a certain temperature their lattice system changes and so do their material properties. The change of material characteristics is especially pronounced for alloys, i.e. metal mixtures. For instance, for an iron-carbon alloy **steel** the physical properties can be changed by hardening and annealing, although the chemical composition is not changed substantially. Also non-iron metals like copper can change their stiffness by bending (strain hardening) although their chemical composition remains unchanged. The root cause again is a change in the lattice structure (lattice defects).

The magnetic material properties also depend on the crystal structure. In the iron atom the electrons *orbiting* around the nucleus produce individual magnetic fields that are externally completely compensated. On the other hand, the magnetic fields originating from the electron *spin* are not completely compensated. Thus, every atom displays an **elementary magnetic dipole**. Inter-atomic forces try to orient the dipoles parallel to each other as well as parallel to the edges of the iron crystal lattice. In a cubic lattice there are six sign-dependent orthogonal edge directions in which the elementary dipoles of a non-magnetized iron crystal are oriented. Accordingly, regions of neighboring atoms with the same magnetization directions are formed. PIERRE WEISS was the first to postulate these regions of equal magnetization direction which were, hereafter, called WEISS domains, elemental domains or simply **domains**. All domains are magnetically saturated; all domain atoms point in the same magnetic direction. In general, a crystallite incorporates many domains. Their individual orientation is statistically uniform with respect to the six lattice orientations. The entire piece of iron is initially macroscopically non-magnetic as a result of this uniform orientation.

Initially the domain walls will move reversibly with application of a very weak external magnetic field. These domain walls are called **Bloch walls**, named after FELIX BLOCH. As a result, the domains that are parallel to the external field will grow. At higher external magnetic field strengths the movement becomes irreversible, i.e. the Bloch walls will no longer return to their initial position after removal of the external magnetic field, but will remain in the nearest energetically favorable level. The movement of the Bloch walls may even lead to a degradation (annihilation) of smaller domains in favor of the larger ones. Even higher magnetic field strengths may lead to the reversible and/or irreversible orientation of the elementary dipoles from the crystal axis direction to the direction of the external field. Once irreversible changes have occurred, the magnetic orientations of the (newly formed) domains are no longer equally distributed and a persistent (permanent) magnetization (**remanence**) will be remain after the external field has been removed.

Permanent magnets are characterized by their excellent resistance of the domain magnetization to external fields. One possibility to achieve this is to reduce the magnetic particles to a size where no Bloch walls may be included and every magnetic crystallite may contain only one domain. In this configuration only the more difficult, less accessible, reorientation processes may occur without the easier movement of Bloch walls. Small magnetic particles may be produced by milling (powder magnets) or by cooling down fused alloys. Alnico magnets belong to the class of **precipitation alloys**, in which magnetic particles can be grown to the right size by an appropriate temperature treatment (annealing).

Alloys are mixtures of materials with metallic properties. For Alnico, the main constituent (the base metal) is iron with additional alloy elements (Al, Ni, Co, Cu). After heating (e.g. up to 1670°C), all of the components are mixed up in a melt which solidifies during cooling. The solidified alloy is single phase (phase = crystal class) at temperatures above 1100°C, which means that it is made up of only one single cubic face-centered crystal class (α). Although the alloy is already solidified, the miscibility of the components is described as the *solubility* which, in this case, means a solid solution. There is, however, a maximum solubility of the alloy components which is temperature-dependent: The maximum solubility becomes lower with decreasing temperature.

The homogenous one-phase mixed crystals that exist at high temperatures dissociate into two new phases which are also cubic space-centered: into the Fe-Ni-Al **matrix** (α_2, basic substance) and into an internally finely distributed Fe-Co phase (α_1). The matrix is only weakly magnetic. However, the ball- or rod-like Fe-Co particles are heavily ferromagnetic. The change of texture from the mono-phase into the double-phase configuration which will evolve during the cool down from 850°C to 750°C is called **spinodal decomposition** (spinodal dissociation) [24]. Electron microscopic investigations have shown that the developing ('precipitated') α_1-particles are located along the cubic edges of the matrix. Once the particles can be magnetized during their development, they can be influenced by an external magnetic field so that they orient in a **preferred direction**. To achieve this, the Curie-temperature has to be decreased by a suitable addition of Co so that it is lowered below the spinodal temperature, because ferromagnetics can be magnetized only above the Curie-temperature. Magnetic materials which have been cooled down in this way in an external magnetic field will show a spatial anisotropy, i.e. their magnetic characteristics are direction dependent. The size of the α_1-particles developed during spinodal decomposition can be changed to a large extent by a several hours long annealing (**tempering**) at 600°C – with substantial influence on the maximum coercive field strength. Most effective are elongated particles with lengths several times their diameter but with sizes well below the onset of Bloch wall generation.

Cubic matrix crystallites with arbitrary orientation are formed during segregation; their edges are pointing in uniformly distributed directions. (For one single crystallite the orientations of the edges are, of course, orthogonal). During cool down in a magnetic field the α_1-particles are arranging predominantly next to the nearest edge orientation, but as the crystallites are still directed in different orientations, the best result is not yet realized. To achieve this, all crystallites in the matrix have to be oriented parallel to each other, which means they have to be grown parallel to the lattice directions. Applying special treatments (unidirectional cooling, homogeneous temperature gradient, quenching plate) it is possible to come close to the ideal situation. Magnets produced in this manor are called *grain oriented, crystal oriented, preference oriented or columnar oriented*. However, they can reach their optimum properties only if the oriented crystal growth (**crystal anisotropy** of the matrix) is combined with a proper magnetic field treatment (**form anisotropy** of the α_1-particles).

In this short excursion into material sciences it should be pointed out that it is not sufficient to simply characterize pickup magnets by their chemical composition. The description of "Alnico V by 8% Al, 14% Ni, 24% Co, 3% Cu" does not provide information on the remanence, coercivity or permeability. Moskowitz [23] summarizes this complex of problems: *There are 16 factors that determine the actual performance of a specific basic magnet in a particular circuit. The magnetic and physical properties of the material are directly dependent on the following factors in the manufacturing process: chemical composition, crystal or particle size, crystal or particle shape, forming and/or fabrication method, and heat treatment. Permeability, coercive force, and hysteresis loop are specifically affected by gross composition, impurities, strain, temperature, crystal structure and orientation. The effects of each of these factors are metallurgically complex and beyond the scope of this book.* After all, "this Book" is called PERMANENT MAGNET DESIGN AND APPLICATION HANDBOOK. This book has most probably not been read by the author of the 2001 published book "E-Gitarren" who wrote: *"The production of a magnet is quite simple. The basic materials will only be exposed to a high electrical voltage ... The field strength of a magnet produced in such a way might be measured in Gauss."* ?? ☹ !!

Magnets with defined magnetic properties *cannot* be produced easily. Contrary to the constant current resistance, magnetic parameters are not easily measured. The resistance variations of ±5% are discussed in depth in pickup literature and the sound difference between Alnico-5 and Alnico-2 is addressed in epic scope. However, the variation of magnetic parameters is usually not mentioned.

Very pure components are necessary for the production of Alnico magnets. McCaig [26] claims iron with a maximum carbon concentration of 0,02%, whereas Cedighian [25] recommends aluminum with a purity grade of at least 99,6%. Moskowitz [23] claims *very close metallurgical controls* and tolerances of, for example, ±0.05% for titanium and ±0.06% for silicon. These tolerances must not only be realized during weighing but also for the melt. Moskowitz [23] demands that Alnico-3 has to be homogenized at 1290°C ±5°C and McCaig writes that a temperature deviation of only 10°C can lead to *extremely poor results*. Did all magnet suppliers apply such a high precision, particularly in the forties and fifties of the last century when the famous vintage pickups were produced? The scientists that were involved in magnet production tried to gain insight into the crystal structures with the microscopes available at that time. However, the optical microscopes could not resolve particles as small as approximately 40nm x 8nm x 8nm. Electron microscopes as well as X-ray equipment were already available, but not in large numbers. McCaig [26] notes: *We at the Central Research Laboratory of the Permanent Magnet Association became interested in the angular distribution of crystal axes in the late 1950s. At this time we did not possess our own X-ray equipment ... Each crystal required an exposure of several hours, so the experiment was not carried out on many samples.* This statement was made in the late fifties. McCaig writes further: *Unfortunately the details of manufacturing processes are rarely sufficient to enable you to produce magnets successfully yourself. Even when a process for making permanent magnets is fully and honestly described, it may take several months for someone skilled in the art to reproduce it successfully in a different environment.* This was at the end of the seventies - and is still valid.

In the early years (decades?) pickups not only had different numbers of turns but also different magnets. **Seth Lover**, developer of the Gibson "Patent Applied For" humbuckers answered to the question whether he permanently used **Alnico-V**-magnets: "We have also used Alnico II and III because Alnico V was not always available. We have purchased whatever was currently available, because they were all good magnets. The only difference was that Alnico V did not lose its magnetization as fast [13]". In contrast to this Gibson's advertisement claims: "*BurstBucker pickups now give guitarists a choice of three replica sounds from Gibson's original "Patent Applied For" pickups – the pickups that give the '59 Les Paul Standard it's legendary sound. ... with unpolished **Alnico II** magnets and no wax potting of the coils, just like the originals*". Right you are, if you think you are ...

"We have purchased whatever was currently available." Obviously, the only important thing was that it was marked "Alnico." However, this name only means that an Iron-Aluminum-Nickel-Cobalt alloy was used. The magnetic properties only develop during heat and, where necessary, magnetic treatments and are manufacturer secrets. One would have to determine the B/H hysteresis to reveal the characteristics of a certain magnet. However, to achieve this, one would have to to demagnetize and remagnetize several times and what owner of a 1952-Les Paul would like to perform such a treatment? Vintage pickups will therefore always be surrounded by a mystical aura.

4.4.1.1 Alnico-III and Alnico-I

Alnico-I was derived from Alnico-III by replacing 5% Ni by Co [21]. Both alloys do not differ significantly in their magnetic properties. Alnico-III is free of Co and, thus, is sometimes called **Alni**. In the USA, however, Alnico-III is assigned to the Alnico magnets, even without Co. **Alnico-I** is mainly used for larger magnets and is not important for pickups. **Alnico-III** was the material of choice for smaller and cheaper magnets – and this is the reason why it was used in the fifties by Leo Fender for the magnets of the Telecaster.

Most of the material science books quote the following composition for Alnico-III: 12% Al, 24-26% Ni, no Co, 0-3% Cu, remainder Fe. The maximum remanence which can be achieved is 0.6-0.75 T, the coercivity is 32-45 kA/m, the maximum energy density is 9-12 kJ/m³. The cool down procedure also has an influence on the magnetic properties, in addition to the chemical composition, and subgroups are designated by additional characters, e.g. Alnico-III-A. Alnico-III magnets are isotropic and are available as cast or sinter magnets.

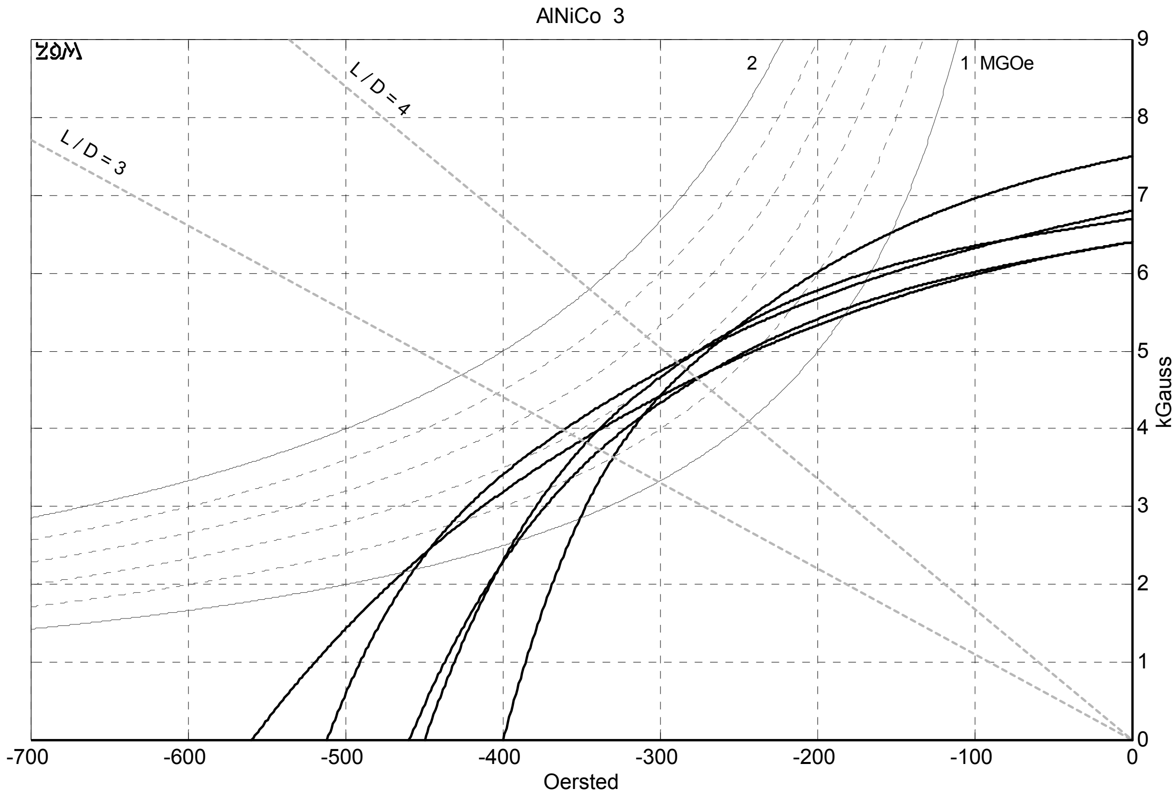


Fig. 4.8: *B/H*-characteristics of different Alnico-III-magnets [21 - 23]. 1Oe = 80A/m, 10kG = 1T, 1MGOe = 8kJ/m^3. L / D = length / diameter (cf. Fig. 4.11).

Fig. 4.8 shows the *B/H*-curves of several Alnico-III magnets. Their points of intersection with the energy-hyperbolas are located close to 1.4 MGOe = 11,2 kJ/m³. The spread of coercivity values, which is depicted as on the abscissa, is considerable.

4.4.1.2 Alnico-II

Alnico-II contains more cobalt as well as copper, which leads to a slightly higher price compared to Alnico-I, -III and –IV magnets [21]. Alnico-II shows the highest BH_{max} value of all *isotropic* Alnicos.

Most material science books quote the following composition for Alnico-II: approx. 10% Al, 17-19% Ni, 12-15 Co, 3 - 6% Cu, sometimes some per mills Ti and S, remainder Fe. The achievable remanence is 0.7 - 0.85 T, the coercivity is 34 - 52 kA/m, and the maximum energy density is 11-16 kJ/m³. In addition to the chemical composition, the cool down procedure has an influence on the magnetic properties. Alnico-II is isotropic and available as cast or sinter magnet. Alnico-II can be treated with external magnetic fields but the gain in energy is only approximately 10% due to its relatively low cobalt content [21].

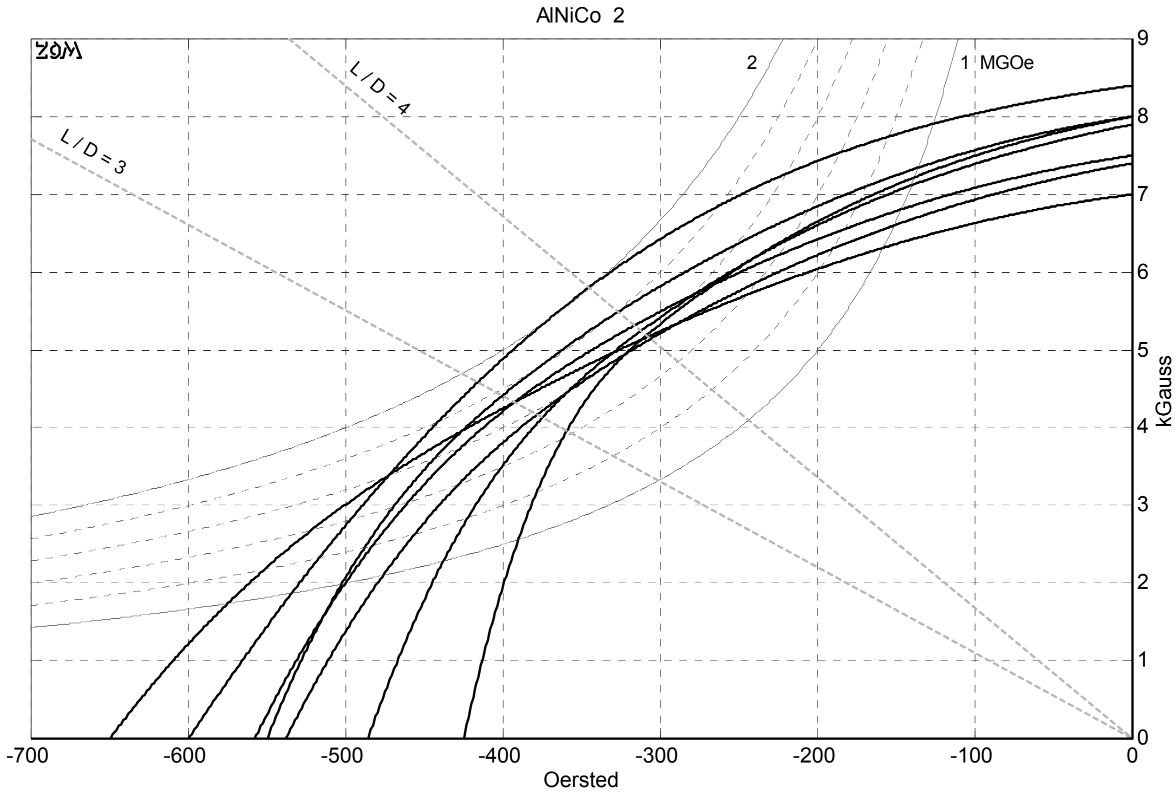


Fig. 4.9: *B/H*-characteristics of several Alnico-II-magnets [21 - 23]. 1Oe = 80A/m, 10kG = 1T, 1MGOe = 8kJ/m³. L / D = length / diameter (cf. Fig. 4.11).

Fig. 4.9 shows the *B/H*-curves of several Alnico-magnets. The maximum specific energy is located between 1.6 – 2 MGOe = 12.8 – 16 kJ/m³. The comparison with Alnico-III yields somewhat higher values for coercivity and remanence.

4.4.1.3 Alnico-V

Alnico-V is anisotropic and reaches the highest BH_{max}-values of all Alnico-alloys [21]. However, its price is higher due to its considerably higher cobalt content. Alnico-V is the material of choice for nearly all Fender pickups.

Most material science books state the following material composition for Alnico-V: approx. 8% Al, 12-15% Ni, 23-25% Co, 0-6% Cu, sometimes some per mills Ti, Si and S, and the remainder Fe. The maximum remanence is 1.1 – 1.3 T, the coercivity is 50-62 kA/m, and the maximum energy density is 30-60 kJ/m³. Besides the chemical composition, also the cool down procedure and the application of magnetic fields has a significant influence on the magnetic properties. Alnico-V is mostly anisotropic and is available as cast or sinter magnet. Alnico-V can be entirely (Alnico-V-7) and partially (Alnico-V-DG) grain-oriented. Many different brand names exist on the international market.

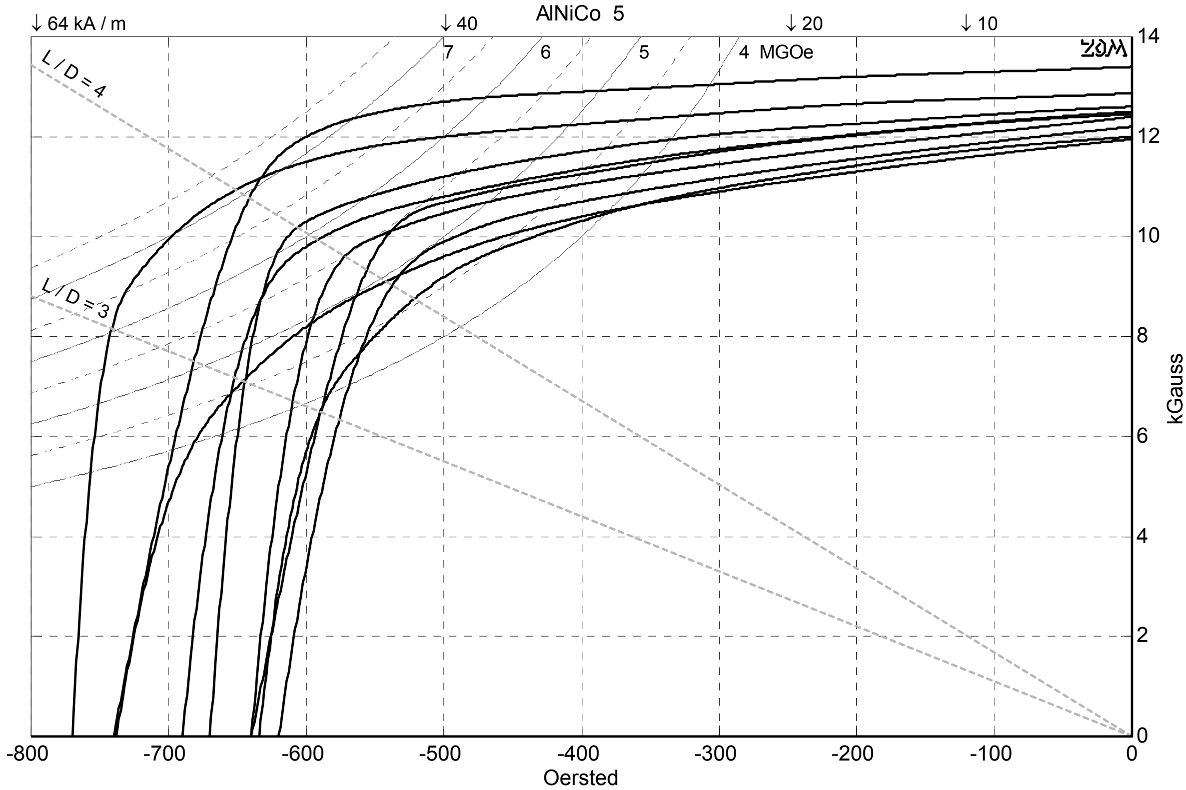


Fig. 4.10: *B/H*-characteristics of several Alnico-V-magnets [21 - 23]. 1Oe = 80A/m, 10kG = 1T, 1MGOe = 8kJ/m³. L / D = length / diameter (cf. Fig. 4.11).

Fig. 4.10 shows the B/H-curves of several Alnico-V magnets. When compared to Fig. 4.8 and 4.9, one recognizes the much more pronounced cubic form of the hysteresis; the maximum specific energy reaches values between 5 – 7 MGOe = 40 – 56 kJ/m³. It can be assumed, with all caution, that the Alnico-V-alloys used for guitar pickups exhibit the lower BH_{max}-values, for cost reasons.

4.4.1.4 Other Alnico-Materials [21]

Alnico-IV has, in comparison to Alnico-I to III, a relatively high coercivity which makes it suitable especially for magnets with a small length-to-diameter ratio.

Alnico-VI was derived from Alnico-V. The coercivity increases with higher Ti content (up to 5%) while, at the same time, the remanence de̲creases. A further increase of this trend is realized with Alnico-VII.

Alnico-VIII, -IX and -XII contain 35% Co. The expensive cobalt enables coercivities up to 130 kA/m, however production is difficult because the material is very brittle. The remanence and specific energy density are smaller than for Alnico-V.

Alnico-V and Alnico-II is used mostly for guitar pickups, occasionally also Alnico-III.

4.4.1.5 Comparison of selected Alnico-Materials

Most guitarists want to play the guitar without considering whether their pickup magnets are crystalline or form-anisotropic. This explains why pickup advertisement does not refer to the material parameters but rather to the sound. The advertizing message sounds more competent with the gleam of expert knowledge and the disclosure of proprietary information. This reads as:

Alnico-II:
"For a vintage-oriented, warm sound. Since the magnetic field is somewhat weaker than for an ordinary Strat-pickup, the string swings out more freely and naturally. The result is an improvement of the sustain behavior."
But also: *"For the rather weak Alnico-II the tone literally breaks down."*
Or: *"Pickups with Alnico-II-magnets are softer in their sound character, posses less treble, are more quiet, more rounded and somewhat less dynamic."*
But also: *"Due to its Alnico-II-magnet, the pickup does not loose treble."*
Or: *"Alnico-2 corresponds rather precisely to a mature Alnico-5-magnet."*

Alnico-V:
"Alnico-V = clear/powerful sound, more wiry twang, more powerful bass."
But also: *"Alnico-V = bluesy base character with pleasantly rounded tone."*
As well as: *"Alnico-V = fast attack and slightly undifferentiated reproduction."*
Or: *"Stronger magnets will deliver less treble."*
But also: *"The stronger Alnico-V-magnet sounds more brilliant."*

Alnico-VIII:
"The higher magnetic power of the Alnico-8-magnet results in a sustain loss."
But also: *"Louder pickups possess more sustain."*
As well as: *"Alnico-8: The pickup produces high output power with little compression also for hard plucking."*

Sources for chapter 4.4.1.7: Gitarre & Bass, Musik Produktiv; Rockinger; E-Gitarren (Day et al.).

Nearly no retailer who promotes pickups in his **advertizing material** makes an effort to investigate differences in sound produced by the exchange of magnets. They may compare two guitars, one of them sounding more trebly than the other, one of them with Alnico-V-magnets in the pickups, the other with Alnico-II-magnets. Then the root cause is clear at once and the advertizing text is ready. The rules of physics sometimes seem to be a real challenge for textbook authors as well:

'According to the information given by manufacturers of magnets, Alnico magnets are supposed not to weaken over the course of the years, but to retain their Gauss-values and thus their magnetic power over a long time. On the other hand, the pickup-industry claims that Fender-type pickups noticeably loose magnetic power already after 2 years, and that Gibson-type pickups do so after 3 years. However, this apparent discrepancy can be explained because the supposed loss in power evidently seems to be a decline in the „retentiveness" of the pickup-magnetism. This means that with the vibrating string disturbing the magnetic field, the particles of older magnets can be more easily thrown into disorder (and thus experience a short-term loss of magnetic force) – compared to brand-new magnets.' (E-Gitarren, Day et al. – German text retranslated into English). Of course, a magnet does have a force – it can draw an iron nail way from a table-top, for example. However this force is measured not in units of **Gauss** but in units of Newton. The unit Gauss relates to the magnetic flux density but this is not the definition applied by the above author-collective: „The field strength is measured in units of Gauss". Sorry, no agreement here – not with the scientific literature, anyway, which defines the **Oerstedt** (in the US) or the A/m (in Europe), respectively, as the unit for the field strength. Day et al. do have a quantity allocated to the unit Oerstedt, as well: "the resilience against demagnetization". That is not completely wrong if we think in terms of the coercive field strength that actually is measured using the unit of Oerstedt and A/m, respectively. However, the term "resilience" again opens the door to mix-ups. "Magnetic power", as well, is such a term that can be misunderstood easily, since power is measured in units of hp, or Watt, or Nm/s. Any author trying to explain difficult technical context with simple, musician-friendly terms runs the danger of being open to attack, and risks to be criticized in case of too rigorous simplification. It does not really help, however, to assign a new meaning to established terms just to achieve the simplification. Of course, a scientist will be criticized just as much if he remains lost in his non-linear differential equations in an effort to maintain exactness and full integrity. Accordingly, the journalist in the German magazine Gitarre & Bass (4/2006) opines: *'Caution, if – in the matter of guitar speakers – somebody brings* ***science*** *to the table. The man*♣ *will probably carry lots of misconceptions. It is in fact best to give such people a wide berth.'* Another statement: *'What's all that scientific nonsense, anyway?'* The same journalist does, however, also write: *'A myriad of these prejudices exist that seem to almost be set in concrete. Who actually decides on such bullshit? These theories are supported by numerous books on guitars written by famous (or infamous) luthiers who actually assume the right to stipulate how much a Telecaster may weigh, or how a Stratocaster pickup should be adjusted.'* And once more a passage from the book "E-Gitarren": *'If pickups remain close to AC-fields such as transformers or strong heat sources, their magnetic structure becomes totally jumbled and they age more quickly.'* O.k. – yes, above 500°C it will indeed start to be a critical situation – but it will not be only the magnetic structure that becomes jumbled, but the tone-generating guitarist's layout, as well: mighty quick aging! *(Paragraph translated by T. Zwicker)*

♣ this would be Dr. Bose, loudspeaker designer and lecturer at the M.I.T. "with dubious formulas"

Obviously, the magnet is involved in the generation of sound: without the magnet there would be no sound. It is also clear that the magnet itself does not have a sound. Alnico-II will not sound different compared to Alnico-V. There is, scientifically speaking, no tone at all if the string does not vibrate. However, one can moan less and talk less elaborately about the *"sound of the magnet"* if one means its effect on the transfer characteristics. So how does Alnico-V *sound*? Different from Alnico-II and, if yes, why?

The *change* of flux density is relevant for the induced voltage in the coil. A strong magnetic field will not induce anything as long it does not change. For a change of the flux density the string has to vibrate in a position-dependent, **inhomogeneous magnetic field**. If the magnetic field would be constant at every position, no voltage would be induced. The inhomogeneity of the magnetic field can be influenced by the magnet material as well as by the shape of the magnet. Replacing the magnet might also change the permeability and, consequently, the resonance of the pickup and/or the damping of resonances by eddy currents (resonance quality). Thus, the behavior is by no means mono-causal, where *one* cause produces *one* effect or rather that every effect can be attributed to *one* root cause. Rather, the relationships are complicated and multi-factorial.

The difficulties start already with the material specifications. Fig. 4.10 shows, that there exist several Alnico-V-alloys. In the pickup literature there are no indications on sub-groups, only "Alnico-V", "The holy grail" or "The originally PAF". Not even Seth Lover was able to tell which material was used during which time period, and how much turns were wound. Was Eric's favorite Paula equipped with Alnico-II or Alnico-V? Unfortunately she is no longer traceable (or rather she is hanging in Japan in 17 safes – and every one of them an original!). Does the transcendental sound of the Roy-B-guitar stem from the Alnico-III magnet or from the fact that vintage Telecaster pickups with resistances beyond 11 kOhm have been spotted? Or maybe it lies with the guitarist?

Fig 4.11 summarizes the **hysteresis-scattering**. In this graphical representation regions were defined based on the trends of the hysteresis curves of many Alnico-materials. One can recognize the scattering and the basic differences. Alnico-II is slightly stronger than Alnico-III but considerably weaker than Alnico-V. In **Fig. 4.11b** the attempt is made to extract a typical single curve from the many different possibilities, but without evidence that these curves are the authentic or the most suitable ones.

When comparing different magnetic materials one first has to define into which magnetic circuit the magnet will be integrated: single coil or humbucker (or special construction types). Single coil pickups with cylindrical magnets, like those that were originally designed for the Stratocaster, do not have ferromagnetic materials other than the magnet. The magnetic load is defined by the shape of the magnet, or rather, strictly speaking, by the shape of the surrounding air space. Frequently the length to diameter ratio is approx. 4 yielding a **working point** near the knee of the hysteresis. **Fig. 4.11** shows two straight load lines for $L/D = 3$ and $L/D = 4$. However, one has to take into consideration that literature values may differ [23, 25], and the slope of the lines is decreased by the neighboring magnets. Very roughly simplified, for pickups with cylindrical magnets Alnico-V will produce a magnetic field twice as strong as that of Alnico-II or Alnico-III. However, the flux density derived from the crossing of the curves corresponds to the center of the magnet (neutral plane, chapter 5.4.1), not to the location of the string.

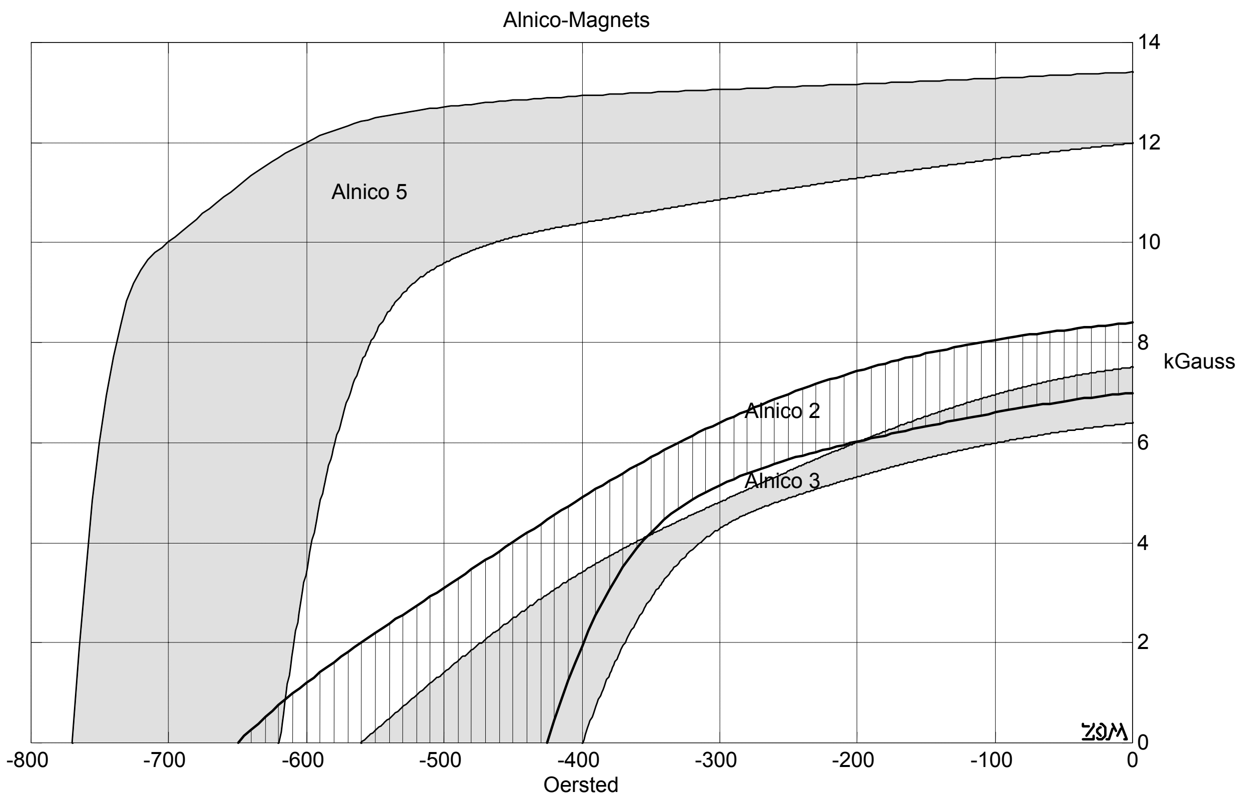
If the magnetic circuit were a linear system this would result in a simple relationship: the vibration of a string would change the magnetic resistance e.g. by 1% and consequently the magnetic flux by 1%. Doubling the static flux, e.g. by exchanging the magnets, would result in a doubling of the alternating flux and doubling of the induced voltage – the generated tone will become louder and possibly more distorted. However, as the magnetic circuit is nonlinear, doubling of the flux will result in an induced voltage slightly less than double. At the same time the magnetic aperture will be decreased (Chapter 5.4.4) and the aperture dependent treble drop becomes weakened, i.e. the pickup sounds slightly more brilliant. An additional brilliance gain with **cylindrical magnets** might evolve from the fact that stronger magnets possess a smaller reversible permeability – the inductance will become smaller, the resonance frequency will increase and the figure of merit will likewise increase slightly (chapter 5.9.3). On the other hand, a treble loss due to eddy currents will also be induced. The electrical conductivity of Alnico-V is approximately 40% higher than that of Alnico-II. It is hard to predict which effect will dominate; however, in most cases the stronger magnet yields a *gain* in brilliance.

The magnetic field at the string location will be weaker for single coil-pickups with **bar magnets** instead of cylindrical magnets. The magnetic aperture tends to be larger and the aperture-dependent treble loss will be somewhat higher. The reversible permeability of the magnet nearly does not play a role because it will hardly be penetrated by an alternating flux. The frequency dependence of the impedance of the SDS-1, for example, will not measurably change if both bar magnets are removed. For the P-90 the magnets have a small influence. They increase the coil inductance by 10%.

The magnets have almost no influence on the pickup impedance for Gibson-type **humbuckers**. The alternating magnetic field passing through them is negligible and, hence, the reversible permeability and the eddy current damping play practically no role. As for the single coil, the magnetic aperture and absolute sensitivity do depend on the magnet strength. The working point of many Alnico equipped humbuckers is located below the hysteresis knee, in a rather inappropriate region. The table depicted in chapter 5.4.1 shows that the static magnetic flux densities of the investigated humbuckers are smaller than most of the single coils.

Mechanical Characteristics of Alnico-Magnets:

Density: approx. 7g/cm³
Hardness: 45 – 60 HRC, brittle, risk of fracture, moldable only by casting and/or sintering plus grinding.
Specific resistance: 0.45 – 0.7 $\Omega mm^2/m$. Alnico-V has a slightly better conduction than Alnico-II. For comparison: nickel-silver = 0.3 $\Omega mm^2/m$, Cu = 0.018 $\Omega mm^2/m$, Fe = 0.1 $\Omega mm^2/m$. Ceramic magnets (ferrites) are, however, insulators.
Reversible relative permeability: approx. 4 – 6; usually lower for stronger magnets.
Alnico has a good corrosion resistance; however, it is not fully rust resistant.
Sinter magnets show a higher mechanical stiffness compared to cast magnets. Their magnetic values are, however, somewhat worse. The quality of cast magnets is also reduced when they have cavities.



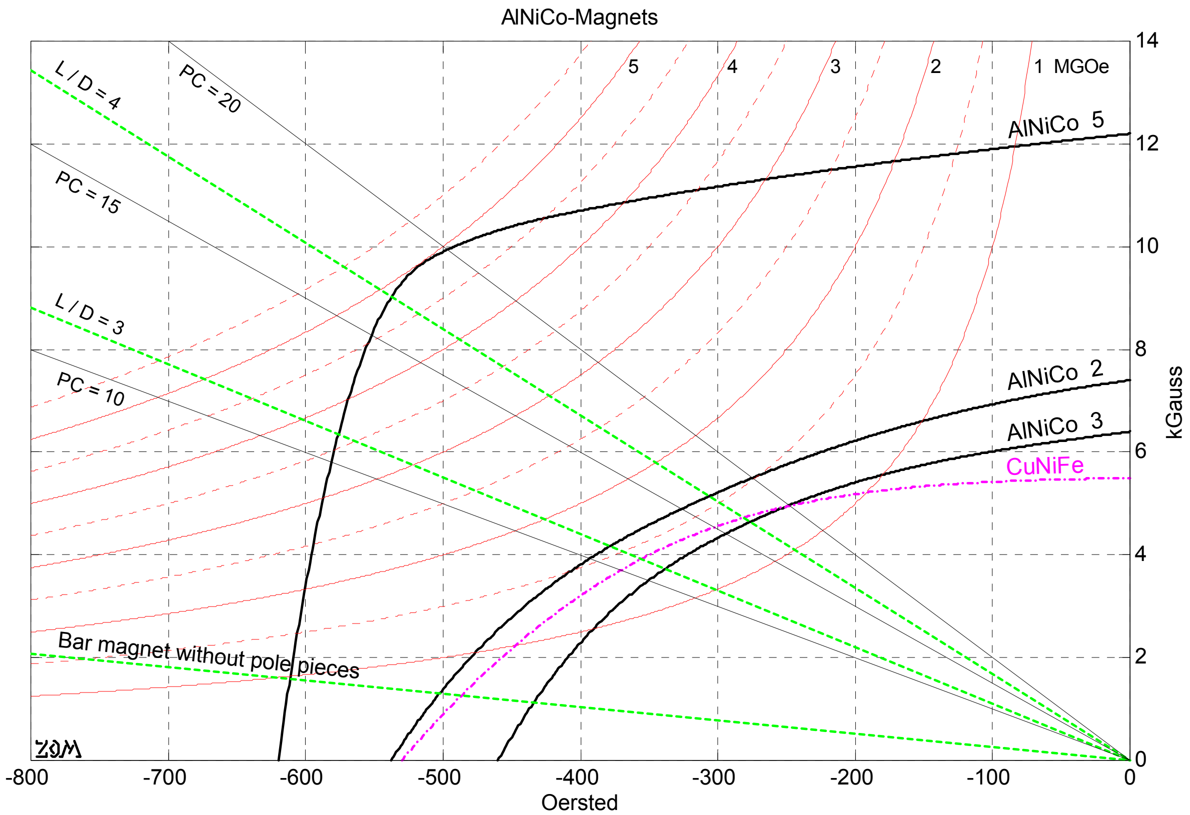


Fig. 4.11: *B/H*-regions of typical Alnico-magnets; the hysteresis curves are located in these regions (upper plot). Lower plot: *B/H*-curves of Alnico cast magnets, data from old specification sheets. 1Oe = 80A/m, 10kG = 1T. L / D = length / diameter. PC = Permeance Coefficient

4.4.2 Cunife-Magnets

Alnico is a very hard and brittle material, which can be machined only with considerable effort. Cutting a screw thread is not possible with ordinary tools. However, this was exactly what Leo Fender wanted when the former Gibson developer Seth Lover built the Fender humbuckers for him: The cylinder magnets had to be adjustable in height by a thread. Cunife, a copper-alloy with an addition of Fe and Ni, which was developed 1937 by Neumann, Buechner and Reinboth in Germany was employed as an alternative to Alnico. The alloy constituents are melted, rapidly cooled and cold-formed. Optimum magnetic parameters are achieved with cold-formed 5 mm diameter wire; this is by chance exactly what is needed as the diameter for pickups with single magnets. The cold-forming yields a heavily anisotropic material with maximum field efficiency in the longitudinal direction. The magnetic parameters are similar to that of Alnico-III.

Cunife (also called **Cunife-1**) consists of 60% Cu, 20% Ni and 20% Fe. The remanence obtained is 5.4 – 5.7 kG, the coercivity 500 – 590 Oe (40 – 47 kA/m) and the maximum energy density 1.3 – 1.85 MGOe (10 – 15 kJ/m^3), which is somewhat higher than for Alnico-III. In addition there is also a **Cunife-2**-alloy with a small amount of cobalt: 50% Cu, 20% Ni, 27,5% Fe, 2,5% Co. This alloy should not be mixed up with Cunico, which has a much higher Co content. Cunife-2 will give higher remanence values at lower coercive field strengths and is, thus, rather unsuitable for pickups.

The big advantage of Cunife is its low **hardness**: The specification sheets in [22, 23] state a Rockwell hardness of B200. However, the B-Rockwell hardness is only specified up to a maximum of 100, so maybe Brinell hardness is meant, instead of the designation 'Rockwell hardness'. The Brinell hardness measurement can only be used for measurements of soft and medium-hard substances and 200HB is characteristic for the lower end of non-hardened steels. The Rockwell Hardness employs a diamond cone (C = cone) and is adequate for harder materials. 45 HRC characterizes the upper end of non-hardened steels, 60 HRC is characteristic for hardened steels. Threads cannot be cut into hardened steel but they are possible in non-hardened steel.

Cunife-magnets have not been widely used. The most famous protagonist is built into Fenders Custom and Thinline Telecasters. It was developed by Seth Lover after he moved from Gibson to Fender in 1967.

Spec. resistance of Cunife-1: 0.185 Ωmm^2/m; Alnico has a 3 – 4 times higher resistance.
Density of Cunife-1: 7.8 g/cm^3, comparable to Alnico.
The relative reversible permeability of Cunife-1 is close to 1, i.e. smaller than for Alnico.

The magnetic properties of Cunife are strongly dependent on the individual production process (cold drawing, annealing), Fig. 4.11 shows approximate values for the *B/H*-curve.

4.4.3 Ceramic-Magnets (Hard Ferrites)

At the beginning of the fifties a new magnetic material was introduced, which is based on the crystal anisotropy of **barium oxide**. This kind of magnet is called a ferrite, oxide or ceramic magnet. Nowadays, mainly **barium ferrite** and **strontium ferrite** are employed. They can be manufactured more cheaply than Alnico-magnets and achieve much higher coercive field strengths, but smaller remanence values.

Ceramic magnets run through a powder-metallurgy production process and their magnetic data can be tuned to a large extent. Their remanence is relatively small at 0.2 – 0.4 T, whereas a coercive field strength of more than 200 kA/m can be achieved. The maximum energy density, of up to 36 kJ/m³, is also much higher than for the Alnico magnets. In contrast to the (comparatively long) Alnico-magnets, a typical ceramic-magnet is relatively short: the optimum length/diameter ratio is close to two. This is the reason why it is employed in (cheap) pickups as a bar magnet beneath the coil, nearly never as cylinder magnet within the coil; for that application the geometry would be too unfavorable.

The relative **permeability** of ceramic magnets does not differ much from 1 and, thus, the inductance of the coil is not increased much, even if the magnet is mounted inside the coil. In contrast to Alnico magnets, ceramic magnets are insulators unable to produce eddy currents. As a result, there is no eddy current dampening of the coil. However, if the field of the underlying ferrite magnet is directed through the coil by iron rods, the eddy current losses are higher as in the case of Alnico cylinder magnet pickups.

Even stronger magnets can be produced with cobalt/neodymium or cobalt/samarium with maximum coercive field strengths of more than 2000 kA/m. These rare-earth-magnets are very expensive – and for pickups only useful in "homeopathic" quantities.

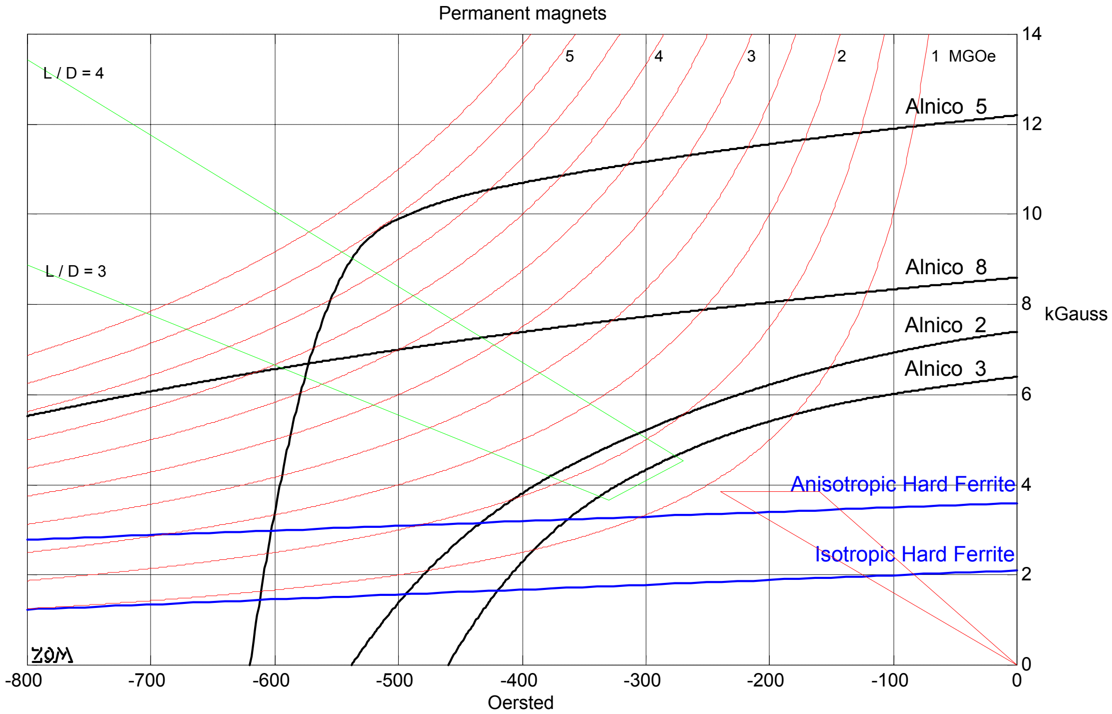


Fig. 4.12: Comparison of Alnico- und Ferrite-magnets. The load curves (different for Alnico and Ferrite) specify the length/diameter ratio of cylindrical magnets (chapter 4.6).

4.5 Magnet-Aging

Provided it is properly made and treated, the life of a modern permanent magnet is, to the best of our knowledge, infinite. McCaig [26] will probably not be able to prove this statement – but also does not have to. Modern permanent magnets will last forever. The magnetic field will decrease measurably only during the first hours following the initial magnetization. At the start some of the magnetic domains are in a meta-stable (unstable or weak) state and rather small energy additions may cause a shift into a more stable energy level. As time progresses these exchange effects will become increasingly less important. To avoid misunderstandings: these processes are called after-effects or aging (ageing, relaxation, magnetic creep, magnetic viscosity, time effect) and not demagnetization. Total or partial demagnetization means the forced shift of the working point to smaller flux values as may be induced by load change or the application of an external field. If a nail attracted by a horseshoe magnet is detached, the flux density will decrease and the working point shifts down to the left on the hysteresis curve (= demagnetization curve) in the 2nd quadrant. This is, of course, not what is meant by aging. If, however, a magnet has lost 5% of its flux density after 10 years of storage without being used, then it has aged. Between these boundaries there is, however, a grey area with components from both worlds.

The main causes for aging are changes of load and temperature; other sources do not play any role for pickups. Reversible aging can be compensated by new magnetization and the magnet then appears "like new". During irreversible aging, however, the internal crystal structure is changed and the former values will not be reached again.

A quantitative description of aging processes needs the application of high-precision measuring equipment and much patience. Prediction is difficult, especially into the future – not different from stock prices. If the flux density has decreased by 0.1% in the first year and the precision of the measuring equipment is of the same order, one cannot make exact predictions for the next 10 years. On the other hand, a measurement covering 10 years is also not without problems, because a great many parameters have to be kept constant during the entire measurement period.

The natural aging without external interference is described by a logarithmic law:

$$B(t) = B_0 \cdot \left(1 - k \cdot \lg(t/\tau)\right) , \qquad t \text{ may not be too close to zero}$$

where *B(t)* depicts the time-dependent flux density, k is a material constant (which can also be dependent on geometry and size) and τ is a reference time, e.g. one day after production. For $t = \tau$ one gets $B = B_0$ the flux density after one day. $k = 0.01$ would mean that B has decreased by 3% after 1000 days. A decrease by 4% would, according to this formula, happen only after 10000 days and a further decrease of B by 1% (to 5% in total) would happen in 10^5 days – which is approximately 274 years. The actual k-values of good Alnico-magnets are still considerably lower, after 10 years typically only 0.1 to 1% is missing. The natural aging thus does not play any role for static magnetic parameters of pickup-magnets♣. Pickup-guru Bill Lawrence is of the opinion that Alnico-5 decreases <5% in 100 years [Billlawrence.com].

♣ Effects on the permeability will be discussed in chapter 4.10.

Temperature changes will result in reversible as well as irreversible changes of flux and field strength. The reversible changes of approx. ± 0.5% are negligible for typical temperature changes. Irreversible changes will occur only beyond +500°C. Some authors, however, quote this limit to be +200°C; every guitarist has to decide whether he considers this as critical.

Load changes in the magnetic circuit (chapter 4.6) and external magnetic fields may, however, lead to dramatic changes. Since the hysteresis loop has two branches, which can only be run through in different directions, successive changes of the field strength $-\Delta H$ and $+\Delta H$ will not lead back to the original working point (**Fig. 4.13**). If, for example, the magnet is removed from a speaker and afterwards built back in again, the flux density and subsequently the efficiency will be reduced from that time on. For pickups, however, a considerable load change (a disconnection of the magnetic circuit) is practically not possible because of the large air gaps involved. They are exposed only to minor load changes by the location and direction dependent magnetic field of the earth (approx. 0.5 Oe = 40 A/m) on one side and by the vibrating string on the other side. Both effects will change the flux density by less than 1‰ so, practically, not at all. In contrast, the magnetic field changes in electric motors are much stronger – and do not lose their magnetization, either, do they? Magnets used in this way are certainly **stabilized**, though, i.e. they are artificially aged. This procedure should actually be applied to every permanent magnet.

During **stabilization** the freshly magnetized permanent magnet will be exposed several times to a field and/or temperature change with absolute values slightly higher than the future specifications. Usually between 10 and 20 load cycles are sufficient. The magnet will loose flux density (e.g. 1 – 5 %), but will become less susceptible to external load changes. After stabilizing, the reversible permeability (and also the pickup inductance) may slightly increase; these details will be revisited in magneto-dynamics (chapter 4.10). If one considers small changes in the lower per cent region, a very fundamental consideration may not be overlooked: the magnetic parameters may vary considerably more due to the production procedures. A scatter of ± 10% is not the exception but the rule.

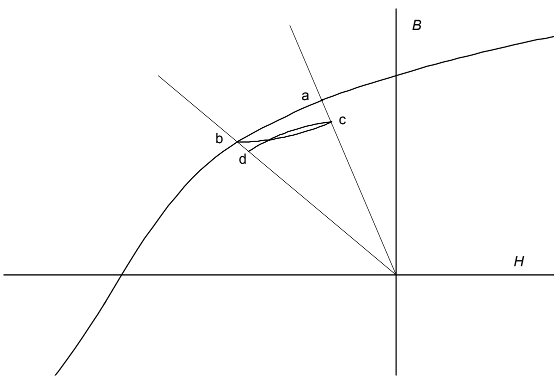


Fig. 4.13: Hysteresis-curve.
Decreasing field strength from point **a** to **b**, and subsequently increasing to **c**, will yield a smaller flux density, i.e. **a** is not identical to **c**. An additional change in field strength will not lead to **b**, but rather to point **d**. After several cycles, however, a lancet-shaped equilibrium state will be reached that will be located slightly below the **d-c** curve. (cf. also Fig. 4.6). The two straight lines represent curves of equal load (Chapter 4.6).

From guitar-literature: a) *"Fender-type pickups will noticeably loose magnetic power after 2 years, Gibson-type ones after 3 years"*. This is physically not justified.

b) *"The magnetizing values of Alnico-2 are matching that of aged Alnico-5 pretty much"*. The forced aging thus would have led to an extreme demagnetization, refer to Fig. 4.11.

c) *"As time goes on, older magnets lose some of their power. The less power the magnets have, the better the strings can vibrate. So maybe after 30 years, the magnets are at their 'ideal' power, thus producing 'ideal' tone."* Guitar collectors beware: Throw away your Les-Pauls and Nocasters from the fifties now – all mag-power lost!

Storage and handling of magnets needs special expertise. If one recognizes in photos, in a professional journal, how the pickup guru has a handful of magnets laying in the drawer, one hopes, of course, that these are non-magnetized blanks, which will be instantly magnetized (behind the kitchen table also shown?) in a super-strong field. Since, if these were bar magnets which are stuck together in a jumble and are mixed up daily, one cannot seriously consider long-term stability and aging (cf. **Fig 4.14**)

Permanent magnets keep their polarization over a long time, but they are not indestructible. Extreme temperatures and force or field impacts may weaken the magnetic field permanently. One should not be worried that a magnet will become weaker when it only falls on a tabletop but, rather, one should be careful with other ferromagnetic materials and other magnets in its vicinity. The working point of an unloaded (open) magnet is located in the second quadrant: negative field strength and positive flux density. If the working point is, however, pushed beneath the "knee", the kink of the hysteresis-curve, the original working point will not be restored again after detachment of the other magnet. McCaig [26] reports on drop impact tests where an Alcomax-III-Magnet has hit a hard wooden floor from a height of 1 m; the measured change was much less than production variation (-0.5 %). In contrast, the magnetic stray field of a second magnet can result in complete demagnetization (-100%).

The following advices may be helpful for the handling of magnets:

- Magnetized permanent magnets should only be shipped with a yoke (keeper). Cautiously remove the keeper before using.
- Do not press magnets with the same poles to each other.
- If attracting parts need to be separated, do not slide them but rather pull them apart.
- The wall of a cylinder magnet should not come into contact with ferromagnetics.
- SmCo- and NFe-magnets may shatter if they collide.
- The attracting forces of strong magnets may possibly be unexpectedly high, resulting in crush injuries.

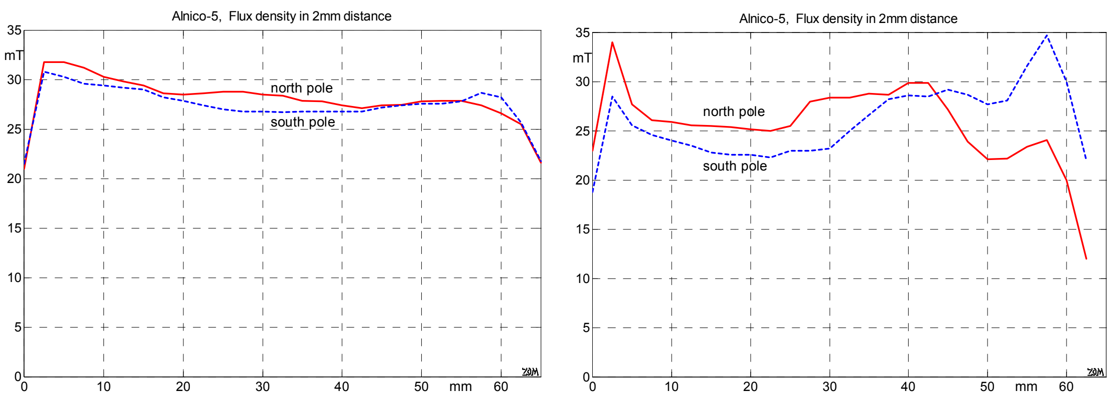


Fig. 4.14: Magnetic flux density measured at a distance of d=2mm along the long side of two humbucker bar-magnets from two different manufacturers. The operating point of the humbucker-bar-magnets is at a rather disadvantageous position and need to be handled with care.

4.6 The Magnetic Circuit

Magnetic fields permeate the whole space (Chapter 4.1). As they are invisible, one depicts their distribution with field lines and performs modeling in analogy to flowing (immaterial) currents. In contrast to electrical field lines, magnetic field lines do not have an origin and an end. As a rule (for which exceptions are possible), they are closed lines with limited length. The tangent to a field line is oriented in direction of the flux density propagation; vertical to it is the penetrated unit area. The corresponding descriptive **field quantities** are the (magnetic) field strength $\vec{H}$ and the (magnetic) flux density $\vec{B}$. The field strength is a length specific quantity (unit A/m) which is determined along its length direction (direction of the flux), the flux density is an area based quantity (Vs/m²), which is specified for the penetrated area.

The line integral along a space curve over $\vec{H}$ yields the magnetomotive force V, the area integral over $\vec{B}$ yields the magnetic flux Φ. If the formula symbol V is already used for volume, the magnetomotive force may also be denoted by V_m. For an infinitesimally small volume element, the differential material quantity **permeability** $\boldsymbol{\mu}$ depicts the relationship between the differential field quantities of field strength and flux density: $\vec{B} = \mu\vec{H}$. In the macroscopic region the integral field quantities magnetomotive force and flux are related by the **magnetic resistance** $\boldsymbol{R_m}$:

$$V = \Phi \cdot R_m = \Phi / \Lambda \qquad \text{Hopkinson's Law}$$

In contrast to the electric field there is no “magnetic insulator”, even vacuum has a non-zero permeability (μ_0). The magnetic resistance is also called the **reluctance**. As the formula symbol R is also used for the electrical resistance, an index m is added sometimes. The reciprocal value of the magnetic resistance is the **magnetic conductance** Λ. Here, no confusion is possible (the electric conductance is Y), so there is no index m. The magnetic conductance is also called permeance and, sometimes, the formula symbols P or λ are used instead of Λ.

A **magnetic circuit** is defined in analogy to the current circuit, which naturally does not have to be a circle. In fact, the magnetic flux flowing (“circling”) around the closed field lines is what is meant. The impetus and cause of the magnetic flux is the magnetic **source**, e.g. a current-carrying wire or a permanent magnet. The boundary of the source is sometimes clearly visible (e.g. the surface of a permanent magnet) but sometimes chosen rather arbitrarily (e.g. a permanent magnet with pole pieces) or does even not exist: The external field of a current-carrying wire runs completely in air, which means outside the source. The part of the flux that is defined as flowing inside the source flows through the **source resistance** (source reluctance), the part flowing outside the source flows through the **load resistance** (load reluctance). In general, each of these resistances consists of sub-resistances, only in the simplest cases there is only *one* source with *one* source resistance and *one* load (**Fig. 4.15**).

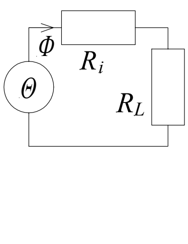


Fig. 4.15: Source with magnetomotive force Θ, source resistance R_i, load resistance R_L and flux Φ (cf. chapter 4.1).

A basic example is the horseshoe magnet (**Fig. 4.16**). Simplifying, one assumes that the magnetic flux is restricted to the magnet itself, to both air gaps and to the yoke (with no leakage flux). The internal, air gap and yoke resistances are successively penetrated by the same flux and are, therefore, represented by a series connection in the equivalent circuit diagram. The magnetic **potential drops** associated with every resistance will, when summed up, result in the magnetomotive force.

A division of the flux into two parallel resistances is necessary if one would like to consider that part of the flux, the **leakage flux**, bypasses the yoke (**Fig. 4.17**). The leakage flux is, of course, spatially distributed and the "channeled" representation in the equivalent circuit is a simplification. If, in addition, one would also like to consider the leakage flux in the magnet, one has to divide up the source (**Fig. 4.18**). This is also a simplification and, if necessary, more than two part sources and more than two part resistances have to be taken into account. **FEM-programs** divide the field structure into thousands of small cells (elements) which are then the basis for the numerical computations of flux and resistance, as well as for more accurate representations of the field lines.

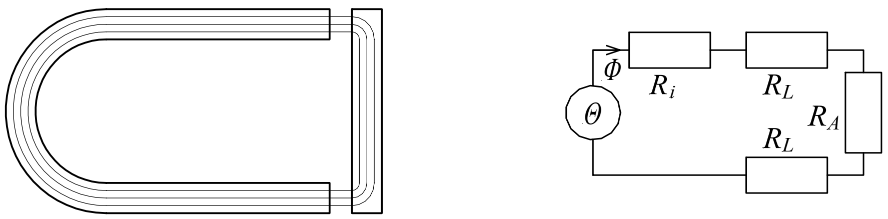


Fig. 4.16: Horseshoe magnet with yoke and no leakage flux.

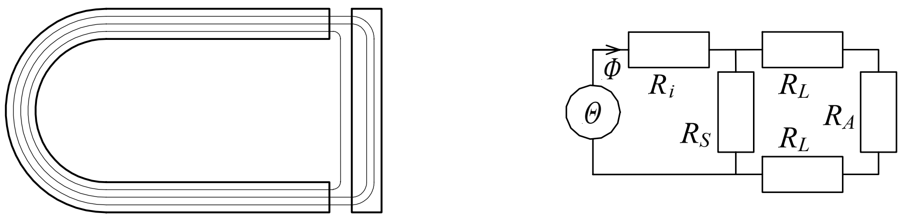


Fig. 4.17: Horseshoe magnet with yoke and load leakage fluxes (schematic).

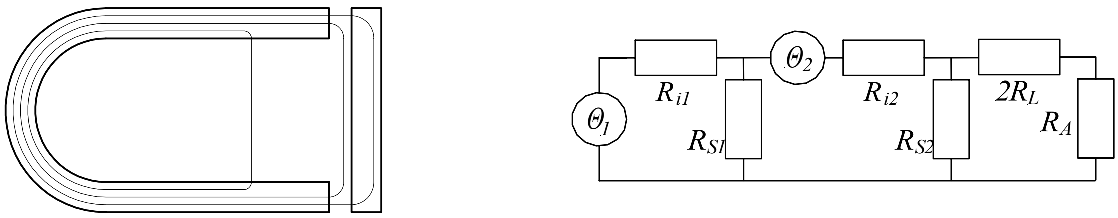
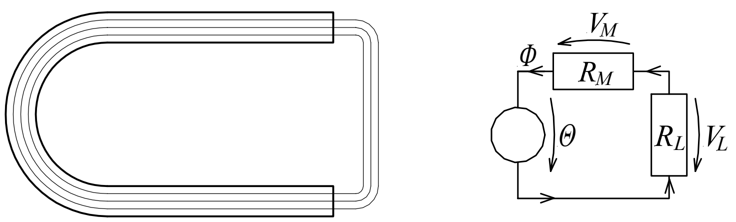


Fig. 4.18: Horseshoe magnet with yoke, source and load leakage fluxes (schematic). The series connection of the both air gap resistances is combined to a resistance 2 R_L.

The determination of the elements of the equivalent circuit of Fig. 4.16-18 is complicated and only approximately possible. For the magnetomotive force, one has to consider the product of coercivity and magnetic length. The source resistance of the magnet is non-linear; it can be determined from the hysteresis. The resistance of the air gap is linear, but it has to be calculated over an inhomogeneous (position dependent) field. The resistance of the ferromagnetic yoke is non-linear. A solution can only be determined iteratively: The spatial field distribution is dependent on the non-linear resistances, but their working point, on the other hand, is field dependent. In particular, it has to be stressed that the, otherwise so powerful, superposition principle cannot be applied here.

However, **Kirchhoff's mesh rule** can be applied, which means in its generalized form: *The flux quantity is equal everywhere in an undivided current circuit; the sum of all potential drops is zero.* When applied to a magnetic circuit this means that, in **Fig 4.19**, the magnetic flux is equal everywhere and the sum of the magnetomotive force and magnetic potential drops is zero: $\Theta + V_M + V_L = 0$. Here Θ is the magnetomotive force, V_M is the magnetic potential drop inside the magnet and V_L is the magnetic potential drop in air. For the sum to be zero, all arrows have to point into the same direction. Alternatively, there are other arrow systems, because each of the three circuit elements can be chosen to have a positive or negative sign. The conventional measurement technique common for permanent magnets yields an unusual sign convention: $\Theta = V_L - V_M$. The arrows of Θ and V_M, on the one side, and V_L, on the other side, oppose each other. In addition, Θ and V_L are assumed to be negative. Two possibilities exist for the flux arrows; it has been defined in such a way that (also unusually) the arrows of Θ and V_L and Φ have the same direction inside the source. This means that, at the air gap resistance R_L, the potential and the flux arrows are opposite but at the magnet resistance R_M the direction are the same. Hopkinson's law is, therefore, written: $V_L = -\Phi \cdot R_L$, and $V_M = +\Phi \cdot R_M$. As mentioned, this needs getting used to.



$$\Theta = V_L - V_M$$
$$V_M = \Theta \cdot R_M$$
$$V_L = -\Theta \cdot R_L$$

Fig. 4.19: Horseshoe magnet without a yoke; no source leakage flux (simplified field line representation).

Hopkinson's law can be represented by a line through origin for the linear air gap resistance; according to the (arrow direction dependent) minus sign, the potential drop is negative for positive flux. The magnet resistance (source reluctance) is non-linear and the V_M / Φ relationship is described by the hysteresis curve. As both R_M and R_L are penetrated by the same flux, both functional graphs can be drawn in the same figure **(Fig. 4.20)**. The sum of V_M and V_L is Θ and this is the distance between the two vertical lines. If one lets the air gap resistance go to zero, as the limiting case, this will yield the remanence flux density. This would be the case when using a very high permeability material as a yoke instead of air. Otherwise, if the air gap resistance tends to infinity the flux density will become zero and one will get the coercivity point. However, a material with $\mu = 0$ may not be realized: no "magnetic insulator" exists.

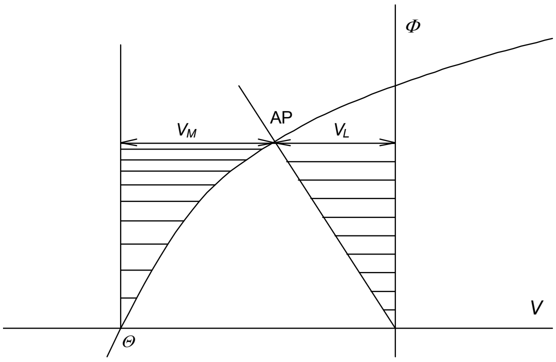


Fig. 4.20: Graphical solution of the non-linear flux equation. V_L increases proportionally to the left as a function of Φ (ordinate!) (straight load line, striped triangle on the right). V_M increases to the right with reference to the ordinate (!) (striped area curve on the left). The sum of both values represents the magnetomotive force Θ at the working point (AP).

Fig. 4.20 contains two functional graphs, which are also called the working characteristics. The non-linear *B/H* or Φ/V relationship is represented by the curved hysteresis line; the linear air gap resistance is depicted by the **load line**. The slope of the load line depends on the individual magnetic field geometry; its intercept with the hysteresis line marks the **working point** (AP). It is only possible to approximately calculate R_L since the air gap field fills the entire (infinite) space.

Cylinder shaped magnets are employed for simple pickup constructions, without any pole pieces (iron parts). Idealized, the magnetic flux will pass through the magnet cylinder in an axial direction, exits the end face, diverging and flowing through the entire air space before it re-enters at the other end face. In reality, however, a considerable source-leakage flux exists: The magnetic flux will also penetrate the lateral cylinder surface which means that the source must be partitioned (Fig. 4.18). The actual, effective air gap resistance, which is often depicted in reciprocal form as the **permeance**, can only be computed by FEM programs with sufficient precision. Indeed, the literature quotes permeance values [21-26] for some simply formed bodies, but their precision is only moderate. This is aggravated by the fact that the unit of permeance can be understood in the American literature only after some deliberation. The permeance *P* is the quotient of flux and magnetomotive force; it should have the unit Vs/A = H or Mx/Gb, respectively. Instead, *P* is given in the American literature with the dimension of length, (e.g. cm), which stems from the incorrect definition of μ_0. The correct value of μ_0 in the cgs-system is $\mu_0 = 1$ G/Oe. Instead, $\mu_0 = 1$ is used, with the consequence, that the units of the derived quantities are also wrong.

In addition to the absolute permeance, a dimensionless **standardized permeance** is also defined, which was introduced by Parker [22] as the **permeance coefficient** *p = unit permeance per centimeter* or elsewhere as a dimensionless *B/H* ratio, again under the assumption that $\mu_0 = 1$. McCaig [26] is more precise and speaks about $\mu_0 = 1$, but means the same. Cedighian [25] again prefers the *B/H* ratio without μ_0 but calls it the **demagnetisation coefficient** (see later). The *B/H* ratio indicates the slope of the load line (load characteristic). The dimensionless designation *B/H* = 12, for example, means that the a strength of 500 Oe corresponds to a flux density of 6 kG. Transformed to MKSA units this would mean that 40 kA/m correspond to 0.6 T. For Alnico magnets *B/H*-ratios of about 15 are optimal in order to run the load line through the point of maximum energy density (B/H_{max}-point). This would yield an optimum length/diameter ratio of about 4 for cylinder magnets if one uses the published permeance diagrams of [e.g. 22, 23, 25, 26,]. In fact, Parker [22] talks about *length-to-area-ratio*, but means *length-to-diameter*.

Many pickup magnets (e.g. Stratocaster) meet this optimum length to diameter ratio quite precisely, but one should not make a dogma out of it; the precision of the permeance data is not very high. The magnetic circuit theory, which is based on circuit analogies, is very well suited to gain insight into the qualitative relations. Nowadays, with FEM computations, more powerful and precise tools are available for quantitative conclusions.

The term **demagnetization** needs a further explanation. First of all, it means every process that shifts the working point away from the remanence point of the hysteresis curve further down to the left – this is the reason why sometimes instead of hysteresis curve one speaks of the demagnetization curve. In addition, demagnetization also means the irreversible destruction of the permanent magnetization, as will occur when the **Curie-temperature** is exceeded or under the influence of a strong field (e.g. inside a demagnetization tool). Demagnetization rarely refers to the aging processes, which are in fact also called aging, after-effect, losses or similar expressions. Rather unexpected, however, is the description of magnetomotive force drops at the source resistance as demagnetization: As R_M in Fig. 4.19 could not be zero, a positive magnetic voltage drop $\Phi \cdot R_M$ will result for every flux $\Phi \neq 0$ at R_M, which - in series with a negative (!) magnetomotive force Θ - will decrease the value of the magnetic voltage drop at the load resistance R_L. The field strength H will also be decreased; the magnetic circuit will thus be "demagnetized". However, one should not force the electromagnetic analogies too far, because otherwise one has to speak about "de-electrification" in conjunction with a charged car battery.

In **Fig.4.21** the load lines are given for typical Alnico curves. A flux density of 9 kG = 0.9 T results for the center of an Alnico-5 cylinder magnet for L/D = 4; if one takes an Alnico-2 magnet instead, B decreases to 5 kG. The standardized permeance values for humbucker magnets are, with $p \approx 4$, surprisingly small; the flux density differences between Alnico-2 and Alnico-5 for are accordingly minor.

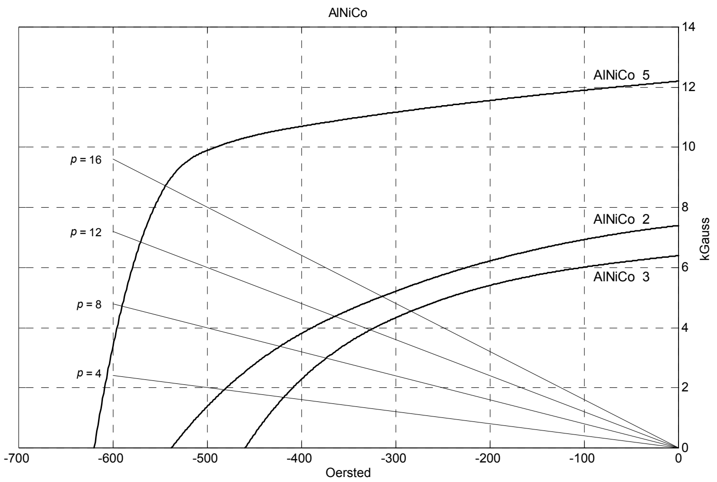


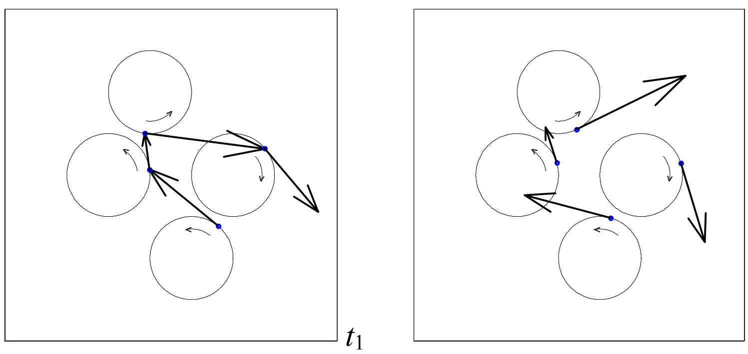
Fig. 4.21: Load lines for different standardized permeance values p = 8, 12 & 16. The flux densities emerging at the working point are achieved at the neutral surface, i.e. at the center of the cylinder magnet. A length/diameter-ratio of approx. 2.5 corresponds to p = 8 for cylinder magnets.
Accordingly: $p = 12 \rightarrow L/D = 3.2$
and $p = 16 \rightarrow L/D = 4$.
Alnico and ceramic magnets have somewhat different permeance values [23].

4.7 The Representation of Magnetic Fields

Magnetic fields are spatial vector fields. Alternatively, to be more precise: the magnetic flux density $\vec{B}$ and the magnetic field strength $\vec{H}$ are vector quantities of a three-dimensional field. In a graphical representation the location, as well as the field quantities, have to be visualized three-dimensionally – impossible on a two-dimensional sheet of paper. Spatial depth can be affected with perspective charts, but distances and angles are very difficult to realize correctly. This is the reason, why cross-sections (and, for special cases, curved areas) are used to draw the field profile. This method is especially suited for parallel-plane or rotational-symmetric fields. For a parallel-plane field, the coordinates only depend on two Cartesian coordinates (x, y). The field of an infinitely long, straight current carrying conductor (Fig. 4.1) is an example if the axis of the conductor is chosen to be the z-direction. However, the same field can also be considered to be rotationally-symmetric with the axis of the conductor to be the symmetry line and with cylinder coordinates (r, φ) instead of Cartesian coordinates. Still there remains the problem of representing the three-dimensional field quantities in this cross-section. Every point in the represented area indeed stands for a point on the cross-section – in fact no space is left to draw the value and direction of the field quantity. So, compromises have to be found and less important information has to be omitted in favor of more important ones. For instance, the field quantity might be depicted only at discrete locations and not throughout the entire (continuous) area. Alternatively, one codes the value of the field quantity by an assigned color and forgoes the direction representation.

4.7.1 Field Strength and Flux Density

The following field representations refer to **two-dimensional fields**. The field quantity is depicted by an **arrow**, whose length characterizes the value and whose direction describes the orientation of the field quantity. Scaling is necessary, e.g. 1 cm ≙ 1 T for the values. The field quantity depicted by an arrow is associated with its root point. This can easily lead to misinterpretations, as the drawing area now has two functions: it represents the position and also the field quantity. The observer is tempted to establish a local relation between the tip and root points of the arrow, although only the root point is assigned to a point on the cross-section. **Fig. 4.22** explains this difficulty with the help of an example of rotating arrows:



t_1 t_2

Fig. 4.22: Velocity vectors, drawn at two different times $t_2 > t_1$. In the left picture a connected polygon line is drawn, however, without physical meaning.

The human brain converts optical information into visual impressions. As a result, the immense amount of data is reduced considerably and structured by **shape** laws. Thus, similar objects in close vicinity are combined into higher-level units with smooth gradients. In the left picture of Fig. 4.22 the tips of the arrows point to the bases of the next arrows in each case. The perceived line is, however, irrelevant, as shown by the picture on the right taken at a later point in time. **Fig. 4.23** shows the upper left vectors of the magnetic field strength. An electric current flows into the image plane at the point [0.5, 0.5], yielding a concentric field. All arrows are tangents to a concentric array of circles; however, due to the large distances it is hard to identify these circles. On the upper right picture the conductor has been moved slightly to the upper right resulting in a giant arrow. This is correct because the base of the arrow is now located very close to the wire and the field strength is actually very high – but this representation is not very clear.

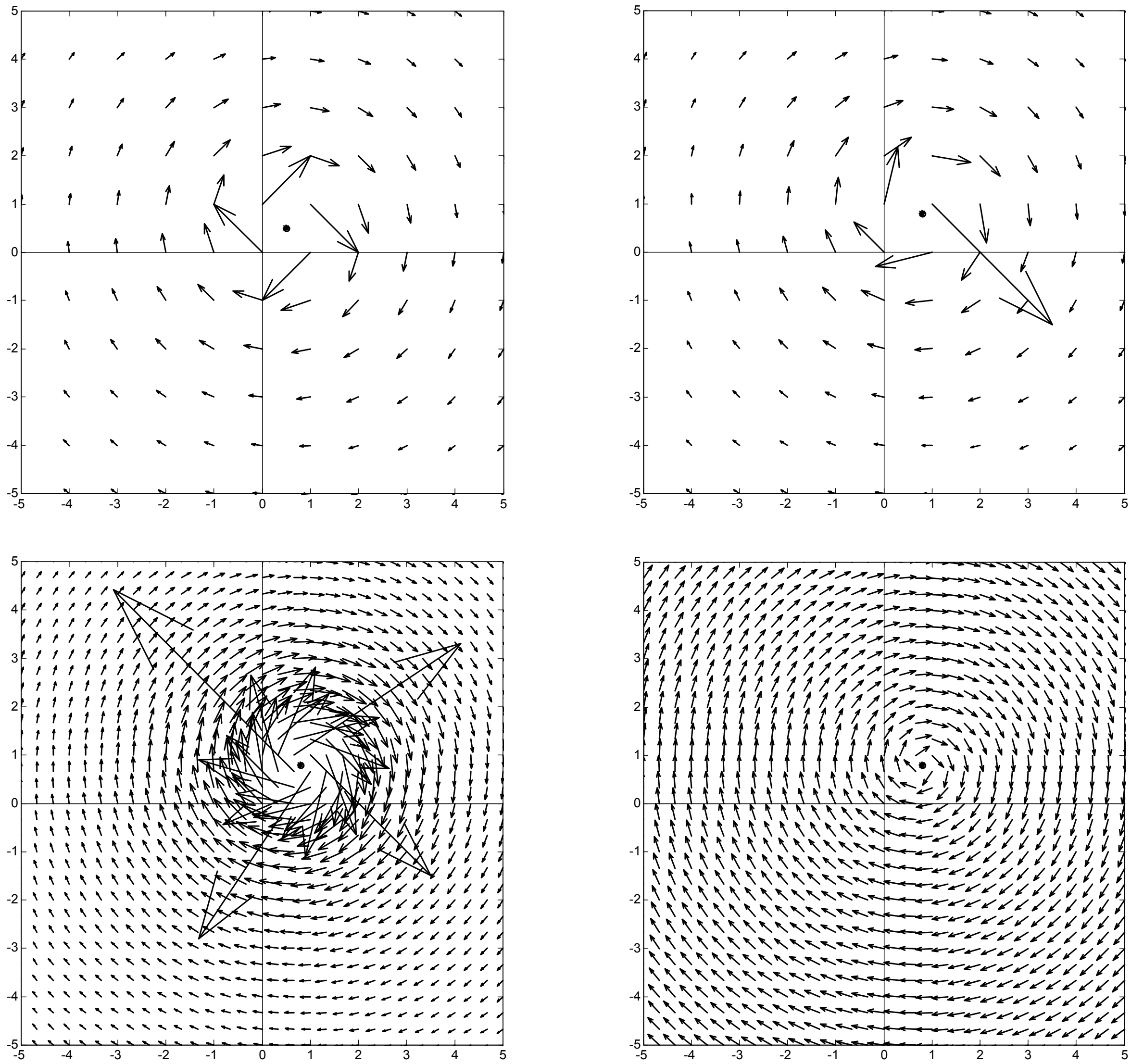


Fig. 4.23: Field strength vectors around a current carrying conductor.

In the lower left picture the density of arrows has been increased in order to increase the resolution of the circles – not a good idea, either. In the lower right picture all arrows are drawn with identical length; the visual impression here is the best but indeed the value information is lost.

If the viewer of the lower right picture of Fig. 4.23 comes to the conclusion that the field is slightly rotated clockwise because there is a slanting characteristic in every picture frame, they perceive an optical illusion: the connection of single arrows to contiguous lines is physically not meaningful for this arrow lattice. Moreover, a rotationally symmetric field cannot be twisted!

Figure 4.24 depicts the field strength vectors of a two-wire conductor. Here, the clarity can also be increased considerably if the value depiction is omitted. If there is a possibility for a color print the value can be shown as colored arrow – on a black and white copy, however, nothing more is visible.

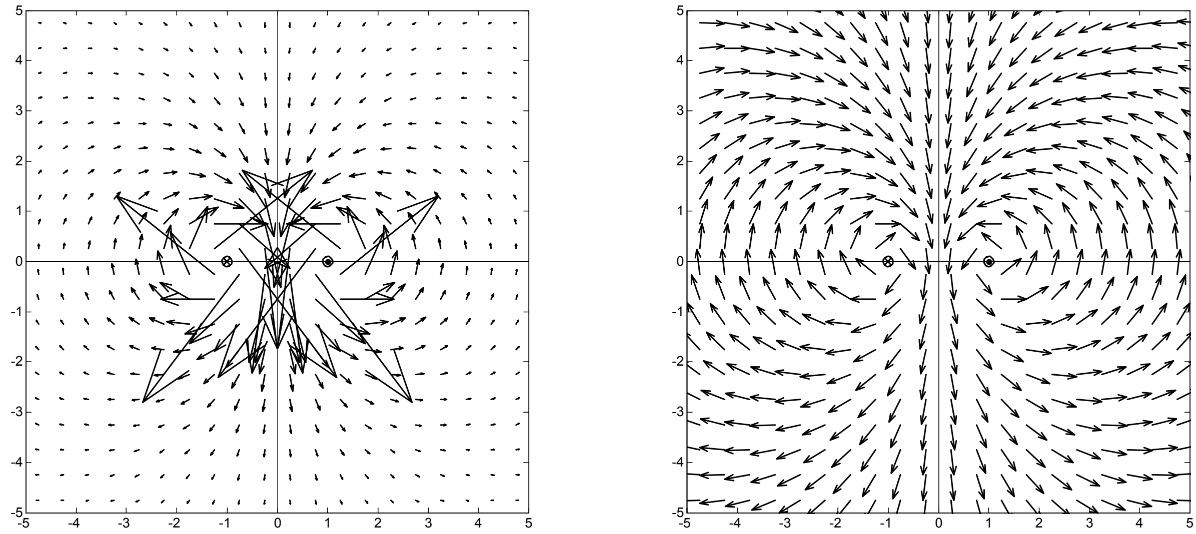


Fig. 4.24: Field strength vectors of a current carrying two-wire conductor (left) and the normalized version (right). In the right image, asymmetries between the upper and the lower part of the picture can be perceived, again based on the misinterpretations shown in Fig 4.23.

In addition to the vector characterization, **field line images** convey a descriptive impression of the spatial field characteristics. Field lines do not show locations of equal field strength – they should not be mixed up with the isobars of a weather chart or the contour lines of a map. Rather a curve will become a field line through the field strength vector $\vec{H}$ defined as tangent vector at every point of this curve. The direction is defined at every point in space as the differential quotient of the field strength. If looked upon geometrically, the integration of this spatial differential equation means the connection of infinitesimally small direction arrows into integral curves, i.e. into field lines (Chapter 4.1).

The field lines of a current carrying conductor are concentric circles. In this simple case one is successful with this equation-analytical description. However, with more complicated real fields, an FEM computation is necessary. **Fig. 4.25** depicts the concentric field: for a current flowing into the image plane the field lines proceed clockwise. The direction of the field strength vector can easily be deduced as tangent to the field lines; however, its value cannot be determined from a field line. Yet an estimate can be deduced from the distance of the field lines: the closer the neighboring field lines, the higher the value of the field. The value is depicted as gray tone on the right of Fig. 4.25, with limited success. The dynamic range that can be presented is not sufficient for a linear relationship; for the 1/r decrease a special color map would have to be defined.

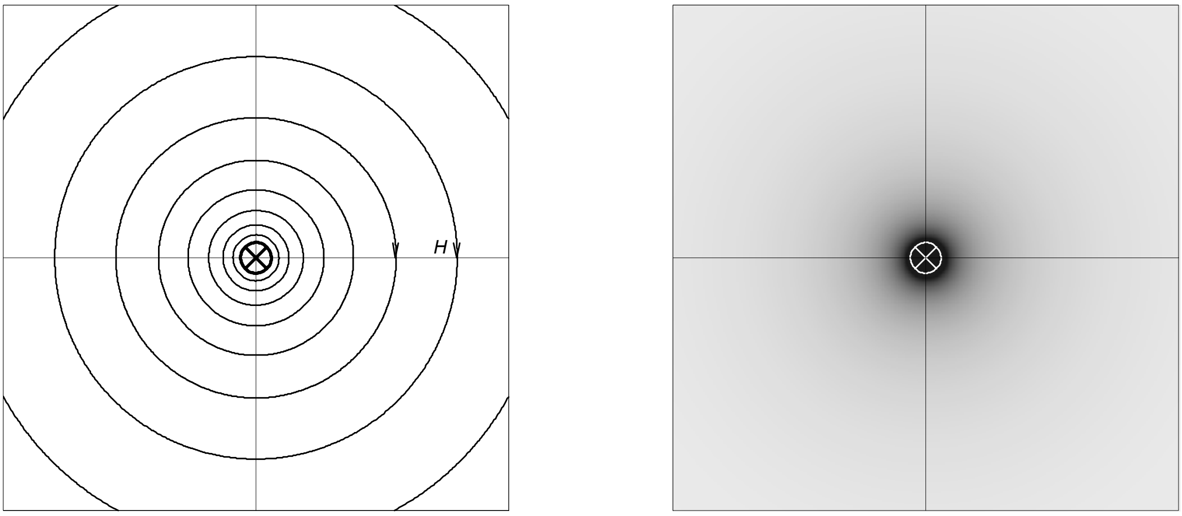


Fig. 4.25: The magnetic field of a current carrying conductor. Left: Field lines. Right: Gray tone coded. The gray tones produced by printers and copiers are not able to resolve the radial field decrease with sufficient precision.

An analytical field-description is also possible for an infinitely long two-wire conductor (**Fig. 4.26**). Assuming opposite current flow, the field lines are eccentric circles with centers located on the x-axis. This could already be inferred from viewing the arrow-description (Fig. 4.24), but in the line description it is obvious. *Between* the wires the field strength is the highest (= maximum line density) and with increasing distance *H* decreases rapidly. A **contour-plot** can be obtained, if all points of equal field strength are connected by lines. This representation is known from maps: A contour line connects all points of equal altitude. However, the expression "contour-line" only means that all points with equal functional values are connected by lines; it has to be specified which value is shown. On an *isobar* the pressure is constant, on an *isotherm* the temperature is constant, for the magnetic field one could call it *iso-field-strength-line*, but this expression is not used. Instead, one talks about **curves of equal field strength**. If one considers curves of equal flux density, sometimes they are called **iso-flux lines**.

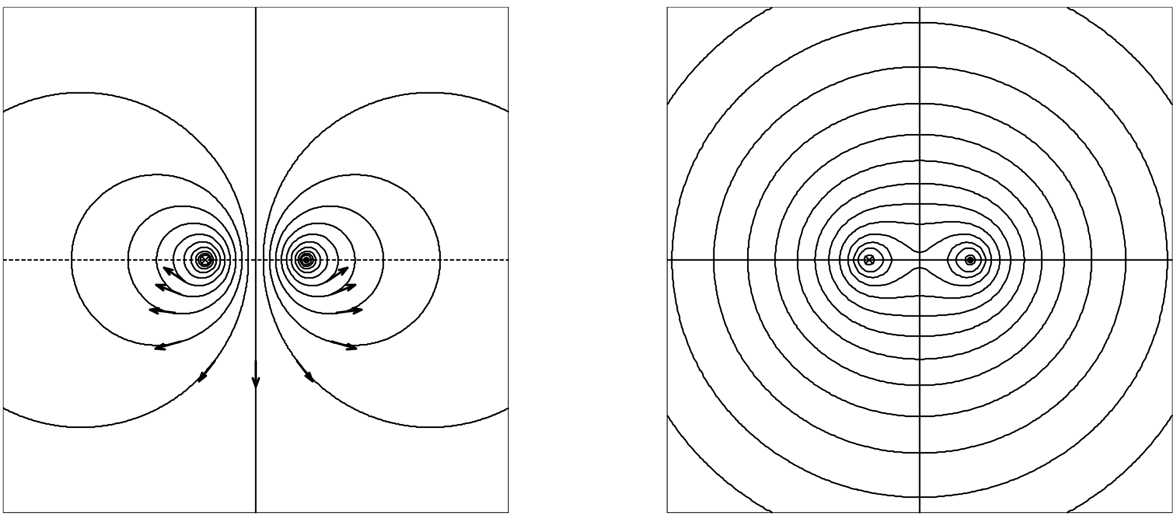
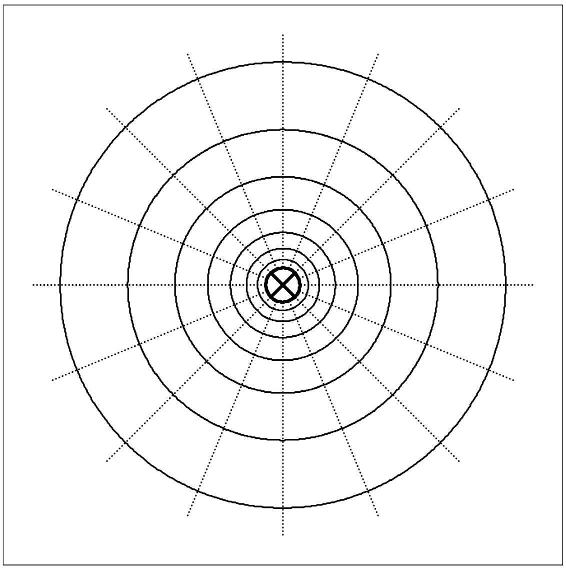


Fig. 4.26: Current carrying two-wire conductor. Field lines with direction arrows (left), contour lines of equal field strength (right). Lines of equal field strength are not equipotential lines (→ Fig. 4.27).

4.7.2 Magnetic Potentials

In chapter 4.2 we introduced the magnetic **scalar potential** and the magnetic vector potential. The negative gradient of the scalar potential ψ is the field strength: $\vec{H} = -\mathrm{grad}\,\psi$. The scalar potential is a scalar quantity that only has a value and no direction. Hence, its characterization (and computation) is simpler than that of a vector. The spatial change of the scalar potential yields the field strength; the line integral over the field strength yields the scalar potential. If one considers very simple fields e.g., that of the current carrying conductor in Fig. 4.25, the field strength is constant on every one of the (circular) field lines. ψ increases proportionally with the angle for rotation with constant angular velocity. Thus, the connecting lines of equal scalar potential values, the so called **equipotential lines**, are rays originating from the center of the conductor outwards. The potential does not change along an equipotential line, perpendicularly it changes the most – this is the direction that the gradient points in. Expressed differently, the field lines and equipotential lines cross at an angle of 90°, the field strength vector is perpendicular to the equipotential line and its value corresponds to the spatial density of the equipotential lines. The field strength is large in areas where the equipotential lines are in close proximity.



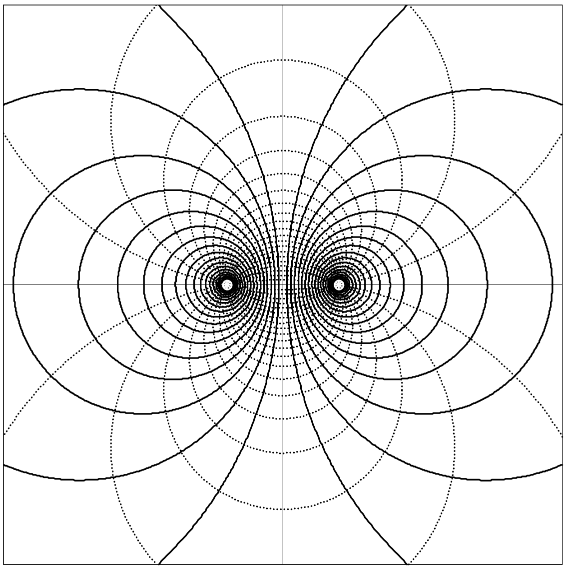


Fig. 4.27: Field lines and equipotential lines of simple fields: single wire (left); two-wire conductor (right).

The term equipotential line depicts curves at which the scalar potential is equal. Considering the **vector potential** $\vec{A}$, equality is much harder to achieve because, as consequence of its vector character, three components have to be equal. The vector potential of a two-dimensional magnetic field, however, has only one component which is normal to the field plane (Chapter 4.2); for a current carrying conductor it is, therefore, oriented parallel to the direction of the conductor. If one defines the field plane as the *x-y* plane, the vector potential consists only of an A_z component, with the partial spatial derivative yielding the flux density vector $\mu\vec{H}$:

$$\mu \cdot \vec{H} = \nabla \times \vec{A} = \mathrm{rot}\,\vec{A} = \begin{pmatrix} \partial A_z / \partial y \\ -\partial A_z / \partial x \end{pmatrix}$$ 2-D vector potential

Considering the **parallel-plane** field, which is conveniently expressed in Cartesian coordinates, a flux density vector $\vec{B}$ can be assigned to every point in space. Both its components are:

$$B_x = \partial A / \partial y, \quad B_y = -\partial A / \partial x, \qquad A = A_z \qquad\qquad A_z = \text{2-D vector potential}$$

The direction of $\vec{B}$ coincides with the direction of the flux lines, $\vec{B}$ is the tangent to the flux line:

$$dy/dx = B_y / B_x \quad \rightarrow \quad B_x \cdot dy - B_y \cdot dx = 0 \quad \rightarrow \quad \frac{\partial A}{\partial y} \cdot dy + \frac{\partial A}{\partial x} \cdot dx = 0 \;\hat{=}\; dA$$

The equation on the right depicts a total differential *dA* which is zero. The total differential can be interpreted as increase in elevation above the *x/y*-plane, if a position change of *dx* or *dy* is made. Since, as discussed above, this increase in elevation for the vector potential is always zero if one runs along a <u>flux line</u> in the *x-y* plane, the value of *A* remains unchanged. Consequently, flux lines are associated with constant *A* values or the other way round: in the parallel-plane field, locations of constant vector potential are connected by flux lines. Hence, for the computation of flux lines (or, after division by μ, of field lines) the vector potential has to be determined and positions of equal vector potential have to be connected.

Cylindrical coordinates are more appropriate than Cartesian coordinates for **rotation-symmetric** fields The computation of the rotation yields somewhat different differential equations to those for the parallel plane field. The corresponding equation for a radius r, rotational angle φ and axis direction z is:

$$\frac{\partial (rA)}{\partial z} \cdot dz + \frac{\partial (rA)}{\partial r} = 0, \qquad A = A_\varphi \qquad\qquad A_\varphi = \text{2-D vector potential}$$

Such a field-symmetry would be adequate for a circular conductor; the vector potential would be circular as well. Flux lines, however, are not positions of A = const, but rather follow $r{\cdot}A = const$.

4.7.3 Spatial Fields

All real fields are three-dimensional and, only for special cases, are they either restricted to thin layers or are (in symmetric cases) characterized by a plane. Field or flux line projections onto a plane are not helpful for the case of a general spatial field evolution – as a rule, the spatial depth cannot be discerned. A last resort would be to define only cross-sections and to depict the flux density or the field strength by color coding within them. The relationship between the represented value and the associated color is given by color maps, which e.g. assign a color gradient from blue to green and yellow to red for increasing functional values.

4.8 Field Distribution in Materials

There is a simple relationship between flux density and field strength in vacuum (or air): $B = \mu_0 H$. In ferromagnetic materials things are much more complicated; here, μ is much bigger than μ_0 and depends non-linearly on H. A larger permeability μ means a higher magnetic conductivity, hence a smaller magnetic resistance. If one assumes that **field lines** (flux lines) always "seek" the path of smallest resistance, one gets a descriptive explanation for the effect of ferromagnetic materials within a field: they "suck," so to speak, nearby field lines into themselves and, thus, encounter – despite the somewhat longer path – a smaller overall resistance. The specialist literature offers yet more comprehensive models, in which e.g. microscopically circular currents are anticipated as the origin of any kind of magnetic behavior. However, for simple cases basic models are completely sufficient.

The simplifying differentiation into magnetic and non-magnetic materials is very convenient for the description of the magnetic behavior of common guitar pickups, because para- and diamagnetism do not play a role. Air, wood, plastics, lacquers, brass, copper, aluminum, German silver (nickel silver) (and many more) are, thus, non-magnetic. Steel♣, nickel and iron are (ferro) magnetic. A copper plate cannot, as some suspect for the Telecaster pickup, "reflect the magnetic field" because, for stationary fields, there is no difference between air and copper. In fact, there is no copper plate underneath the Tele-pickup but rather a *copper-plated* steel plate, which was also sometimes tin-plated, zinc-plated or something else – that is a different story. Ignoring the thin copper layer and assuming a ferromagnetic behavior for the rest means that we have a magnetic material located in the field of one (or several) permanent magnets. How does this change the field profile and what is the effect of this material?

Ferromagnetics conduct magnetic fluxes better than air, comparable with a drainage pipe in the soil that should drain its vicinity: "Mühsam sind des Wassers Wege, in der Erde fließt es träge, doch in Rohren aus Schamott, rauscht's von hinnen mächtig flott♦." Drainage pipes have a lower flow resistance than soil, and this is similar for magnetic fields: The flux density is increased inside the ferromagnetic material and is decreased in the neighboring air. In **Fig 4.28** the field-focusing effect of a ferromagnetic material is depicted for three different permeability values (= conductivities): the higher μ is, the larger is the effect. However, it is not unlimited: even for a very high magnetic conductivity, the effect on the outermost field lines is marginal – they are already too far away.

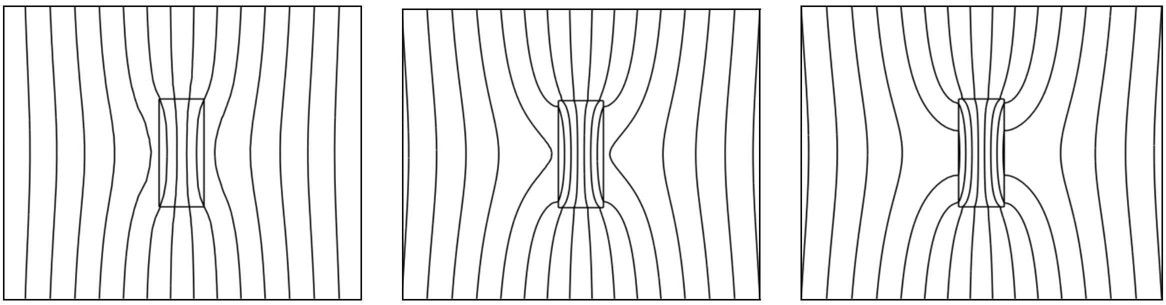


Fig. 4.28: Field line patterns in ferromagnetic materials for stationary parallel plane magnetic fields with no eddy currents. For simplification, the permeability is chosen as locally constant: μ = 5, 50, 5e6 (left to right).

♣ Some steel grades are non-magnetic ♦German rhyme (no, not by Schiller) – very roughly translated: "Arduous is the water's path, in the earth it runs sluggishly, but in pipes made of clay, it very quickly runs away."

The efficiency of the field enhancement (or flux enhancement) of the ferromagnetic material is described by the **permeability** μ (chapter 4.3). There are already clear differences between $\mu = 1$ (e.g. air) and $\mu = 5$ (Fig. 4.28), also between $\mu = 5$ and $\mu = 50$. However, with increasing μ the visible gain becomes smaller and, even with $\mu = \infty$ (compared to $\mu = 50$), there is no further dramatic change. **Fig 4.29** illustrates this with an example: As long as there remain magnetic resistances (air-gaps) in front of or behind the ferromagnetic material, field lines escape into the air space.

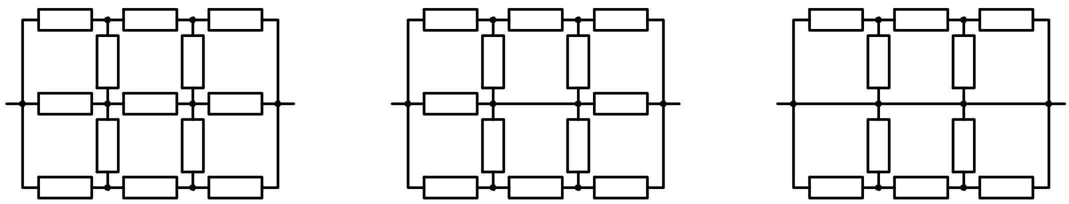


Fig. 4.29: Electrical current flow through a locally discrete model. A current still flows in the outer branches even when the central resistance is decreased to zero, (middle picture). The outer branches are only free of current flow for a complete short circuit (right picture). Analogy: electrical current ↔ magnetic flux.

The above assumed limiting case $\mu = \infty$ in fact is not achievable, but it is helpful for boundary layer considerations: under this limiting case the field strength within the metal turns to zero, because B cannot become infinite. From the continuity condition of the boundary-parallel tangential field strength it can be deduced that the air field strength along the metal boundary also has to be zero. According to $B = \mu \cdot H$ the flux density in air also has to be zero. The immediate vicinity of the (lateral) boundary layer is, therefore, approximately magnetic field free. The quantitative value of the flux density, e.g. in Fig 4.28, can be visualized by the distance between neighboring **field lines**: The denser they run, the larger is B. Since it was assumed that air is the medium around the ferromagnetic material in Fig. 4.28, regions of high flux density (above and below the square) are also regions of high field strength. Accordingly, beside the square, there are regions of relatively low flux density and field strength. It is arbitrary which figurative density of field lines (lines per centimeter) one associates with a particular value of the flux density and it is dependent on the line width, the print quality and angle resolution of average eyes. With only 1 line per cm one may give away space, with 100 lines per cm one may overburden printer and observer. Scaling hints in the picture (e.g.10 lines/cm $\hat{=}$ 1 T) may be helpful, but were left out in the figures because only the relationships are of interest here. If one splits the field vectors at a **material interface layer** (wall) into a wall-parallel (tangential) and a wall-normal component, it follows:

The wall-parallel field strength as well as the wall-normal flux density is always steady. If the permeability on the one side and the other is different the wall-normal field strength and the wall-parallel flux density are unsteady.

For the general case, i.e. for varying permeability, every field line will form a kink at the material interface layer – it is broken, like a ray of light. The larger the difference of both permeability values, the larger the kink. For the angle (α) to the normal of the interface and the tangential flux density B_t this yields:

$\mu_2 \cdot \tan\alpha_1 = \mu_1 \cdot \tan\alpha_2$ $\mu_2 \cdot B_{t1} = \mu_1 \cdot B_{t2}$ Interface conditions [7]

Generally, the permeability of ferromagnetic materials is large and so the field lines will exit approximately vertical from them, i.e. normal to the wall.

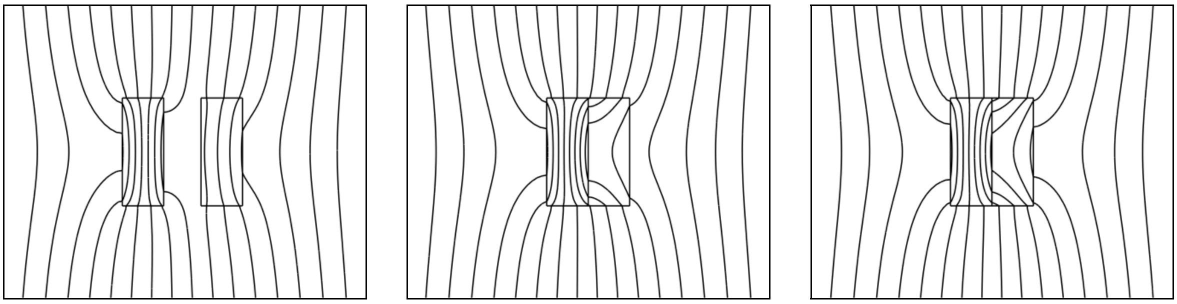


Fig. 4.30: Parallel magnetic conductors; μ = 500 / 5 (left), μ = 500 / 5 (middle), μ = 500 / 50 (right).

Fig. 4.30 shows how two magnetic conductors influence each other. If positioned parallel in the flux direction, each conductor bends the course of the flux density not only in the surrounding air, but also in the neighboring magnetic conductor. The material of higher magnetic conduction (left in the picture) diminishes the magnetic flux in the material with lower conduction and, thus, forms a sort of shielding. The magnetic conductors act on each other as flux enhancing when positioned in series with respect to the flux (**Fig 4.31**): the magnetic conductor placed in front of the other seems to concentrate the magnetic lines like a "convergent lens".

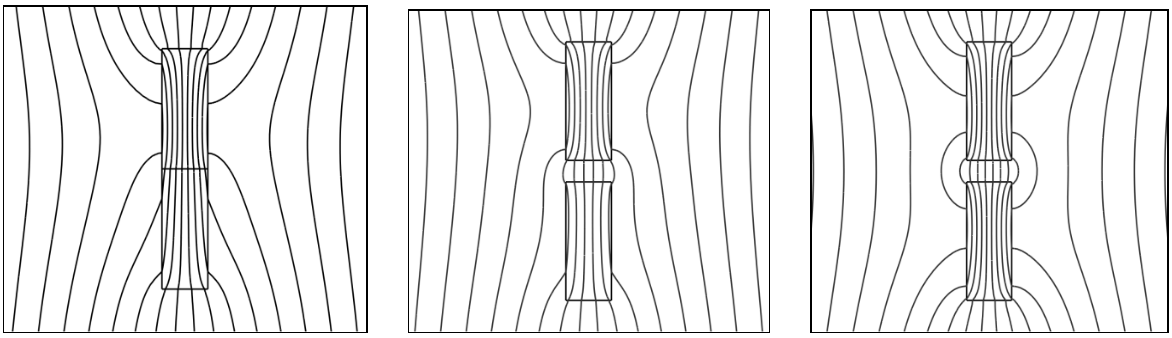


Fig. 4.31: Serial magnetic conductors; μ_{above} = 500, μ_{below} = 5, 5, 5e6 (left to right).

If two materials with good magnetic conduction are placed together, one has to pay attention to the **air gap** in between them, especially in serial configurations: because of the possibly large differences in permeability, very small gaps may lead to considerable magnetic resistances. For pot cores, air gap clearances of e.g. 0.1 mm have to be met with a precision of only a few µm.

A constant **permeability** was used for the calculations of Figs. 4.28 and 4.31, which may be permitted for introductive treatments. However, accurate analysis will need precise material parameters and, consequently, the calculation effort will increase. In fact, there is a mutual dependency between flux distribution and conductivity: high conductivity will yield high flux density but this will also (as a function of the declining hysteresis curve) lead to a decreasing conductivity. This will yield a lower flux density and in turn a lower conductivity – it is a very complicated coupled system.

Non-linear FEM-models approximate this iteration process by many (sometimes even very many) calculation steps, which may occupy a PC for half an hour, or maybe even longer, according to the complexity of the task and the performance of the computer. In addition, not every material is magnetically isotropic. In fact, for a single crystal, magnetic **anisotropy** is the rule and not the exception. If a magnetic field is applied pointing in one of the preferred directions, the magnetization energy is lower than for the other directions. Homogenous spatial distribution of the magnetic crystallite orientation may lead to isotropic (non-directional) macroscopic behavior, but the crystallite orientations are not always uniformly distributed. In fact, for grain-oriented Alnico-5 magnets a special **anisotropy** is perfected, at pole pieces and/or strings it can happen more or less accidentally as a side-effect. Fig. 4.30 has already shown that field lines in a material may run in very different directions. For anisotropic substances the material parameter tensors would have to be specified for FEM calculations, and they are often not available with sufficient precision.

As long as a parallel-plane field is assumed, like in Fig. 4.28 to 4.31, the computational effort can be limited, to some extent, because one can calculate with plane mesh elements. Rotational symmetry may also reduce the calculation effort but, for general 3-D models, things become elaborate. Yet, this is exactly what is necessary for the computation of the magnetic field of a pickup. All these challenges, such as non-linearities, inhomogeneities and anisotropies, impede the calculation, but they do not prevent it entirely: The stationary flux can be determined with sufficient precision. However, the real challenge is yet to come: the magnetic flux which is important for the induction voltage is the alternating flux, i.e. the time-dependent change of the steady flux. This part of the flux is only about 1% of the steady flux! An FEM calculation with “only” 2% accuracy will suddenly lose its original attraction.

On the other hand, if only the basic magnetic flux characteristics need to be described, e.g. for qualitative considerations, FEM calculations are mostly helpful tools. In **Fig. 4.32** a U-shaped metal bar within a magnetic field is shown. A similar arrangement can be found for humbucker pickups (Chapter 5.2, 5.7), with a central bar magnet and adjacent pole pieces. The left picture immediately shows that the humbucking effect might not be sufficient – even though the magnetic flux representation may quantitatively not be drawn with particular precision.

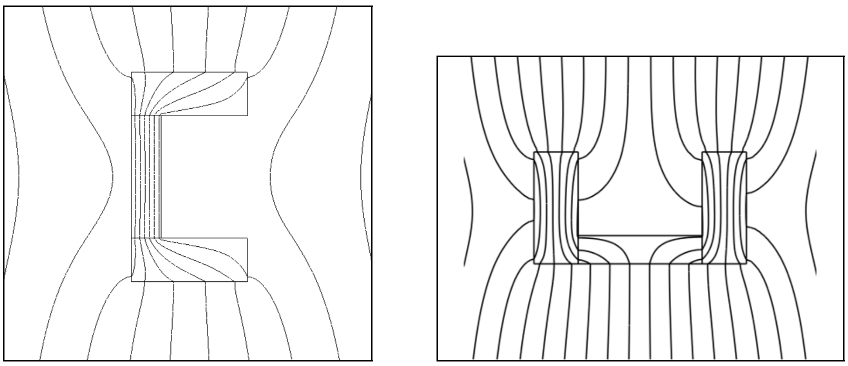


Fig. 4.32: U-shaped bar.

It is hardly possible to represent the magnetic fluxes of a guitar pickup with sufficient precision. The calculation is very elaborate, because one has to deal with non-linear, inhomogeneous, time-dependent and unsymmetrical fields. Further, measurements are difficult, because the lateral dimensions are so small: the diameter of the treble strings is only a few tenths of a millimeter. So, one would need very small Hall-generators which are moved by micro-manipulators on defined paths to determine the spatial extension of the fields. The measurements presented in chapter 5 are, presumably, the first of their kind, but surely not the most precise ones. This makes a basic insight possible, but the differences between similar pickups cannot be deduced from this data yet – to improve, one would need measurement tools which are beyond the current college budget.

4.9 Mathematical Field Theory

A field is defined as a special domain in which a physical quantity is depicted as a function of space and time. Mathematics supports the analytical field description by field and vector analysis as well as by complex function and transformation theories. The following gives a short overview of these theoretical field descriptions; detailed information can be drawn from the books of e.g. Bronstein, Papula, Smirnow, Heinold/Gaede.

The general field theory is powerful, but complicated. Simplifications are possible at the expense of generality which, in many cases, results in practically no restrictions with respect to precision. Magnetic fields propagate with the speed of light (~300000 km/s). The distances of approximately 10 cm relevant for pickups will, thus, be completed in 0.3 ns. Or the other way round: The **settling times** of the fields are much shorter than those that are typical for audio applications (µs – ms); this is why the time needed for the field set-up can be neglected. Simplifying, one assumes that the field change in the entire area takes place at the same time (phase-synchronous), that the magnetic field is free of memory effects, and that the current change of the field state is caused only by the actual excitation, not by spillover from the past. These kinds of fields are called quasi-static or quasi-stationary. A **quasi-static field** is in equilibrium and no energy transport or energy transfer is taking place. A **quasi-stationary field** is in a time-independent state of persistence, including energy transport or energy transfer. The prefix *quasi* stems from the fact that the time derivative $\partial/\partial t$ is *virtually* zero.

The magnetic field is described by both the vector field quantities $\vec{B}$ and $\vec{H}$. A vector field is called **conservative**, if the **rotation is zero** in the region under consideration; in this case the line integral is only dependent on the start and end points of the integration line and not on the integration path itself. Conservative fields are also called **potential fields**, because they possess a **scalar potential** ψ and the vector field quantity is the gradient of the scalar potential, which is the reason why it is also called a **gradient field**. The magnetic field is conservative only in the regions without electrical current flow. In addition, these regions have to be simply connected (Fig. 4.4). In mathematics a gradient field is often described by $\vec{x} = \operatorname{grad} \psi$, with $\vec{x}$ being the vector field quantity and ψ being the scalar potential. The gradient (of the scalar ψ) is a vector pointing into the direction of the largest field increase. However, for the magnetic field the respective formula contains a minus sign: $\vec{H} = -\operatorname{grad} \psi$. The field strength $\vec{H}$ of the magnetic field points into the direction of the largest potential decrease.

The spatial characteristics of the field vectors (e.g. $\vec{H}$) can be visualized by **field lines**. A curve $C(x,y,z)$ is a field line if the field vector at every point of the line is a tangent vector: $dx/H_x = dy/H_y = dz/H_z$. The solution of this differential equation yields the spatial vector field $\vec{H}(x,y,z)$. The field lines do not cross, except for the points at which $\vec{H}$ is not defined or becomes zero. For the magnetic field it can be useful to differentiate between field lines (originating from $\vec{H}$) and **flux lines** (originating from $\vec{B}$); flux lines sometimes are also called stream lines. Here, the literature does not have a consistent terminology.

The flux lines of a magnetic field are (as a rule) closed curves – without a beginning and an end. This is the origin of the expression **source-free field**. Obviously a natural impetus exists which is, however, not the origin of the flux or field lines. Within the framework of vector analysis, source-free means that the **divergence is zero**. Such a field sometimes also is called **solenoidal**. However, $\text{div}\vec{H} = 0$ is not valid for every magnetic field; if ferromagnetics are incorporated, $\text{div}\vec{H}$ can be non-zero because of the field-dependent permeability.

The expression flux line implies that something is flowing (streaming). **The flux** (the stream) can be objective (e.g. water circuit), or abstract (e.g. magnetic flux). As we just have explained, the field changes propagate with the speed of light, so it would appear obvious that the flow-rate corresponds to the speed of light. But one has to distinguish between the flow-rate and the velocity. If one throws a stone into the water, a circular wave propagates on the water surface. This, however, doesn't mean that all the water molecules move outwards with this velocity. The wave propagation velocity is much higher than the **particle velocity** which, for distinction, is called the *particle velocity*. The entire water surface would elevate and drop in phase for an infinite wave propagation velocity, without a visible wave pattern. The time differential of every particle displacement is the particle velocity – which also would be in phase across the entire water surface. The flux has the dimensions volume/time for flowing water whereas the area specific flux density has units of length/time, which might be interpreted as velocity. The dimension of the magnetic flux is Weber or Volt-second, the dimension of the flux density is T = Wb/m² = Vs/m². Here, it doesn't make sense to strenuously search for the dimension m/s. In fact, one might draw **analogous conclusions** on the basis of isomorphic (with identical structure) network graphs and the corresponding equation systems between magnetic circuits and water circuits (and other flow circuits). At the same time it should be clear, that the magnetic flux density does not correspond to the constant wave propagation velocity, but rather to the signal-dependent particle velocity. As the last analogy example we consider the **sound field**: the velocity of sound is constant at approximately 340 m/s. The velocity of the air particles is very much smaller – depending on the excitation. The sound flux q is derived from the particle velocity by multiplication with the area, not from the velocity of sound.

Within the framework of the above explained analogy considerations, $\vec{B}$ can be interpreted as a vector flux field, whose **flux integral** yields the flux Φ:

$$\Phi = \int_S \vec{B} \cdot d\vec{S} \qquad \Phi_q = \oint_S \vec{B} \cdot d\vec{S} = 0$$

Flux integral over S

The flux integral is the area integral over the area S. In case S is a closed, enveloping surface, as it is for the integral on the right, the flux integral describes the source flux emanating from the enveloping surface. As the magnetic flux lines do not have an origin and an end, it is clear that the source flux emanating from the volume limited by S must be zero; the field is source-free or solenoidal. If the envelope surface and the enclosed volume tend to zero, one obtains the divergence:

$$\text{div}\vec{B} = \lim_{V \to 0} \left(\frac{1}{V} \oint_S \vec{B} \cdot d\vec{S} \right)$$

Divergence

The divergence is applied to vectors; the result is a scalar that provides information on the **solenoidal strength** at that position. Vector fields whose divergence is zero are called **solenoidal**. If one forms the line integral rather than the area integral on a vector field value, one ends up with the **contour integral**:

$$W = \int_s \vec{F} \cdot d\vec{s}$$ Work integral along the curve s

If $\vec{F}$ depicts a force, then the line integral results in the work performed (= force · path). Within the gravitational field the work integral only depends on the start and end points, not on the path taken. The value is zero in the gravitational field if the contour integral is computed over a closed line s (starting point = end point),. These fields are called **curl-free** fields. In analogy to the divergence, the curl (**the rotation**) can be depicted as the limiting case of an infinitesimally small circulation path:

$$\mathrm{rot}\vec{H} = \lim_{S \to 0} \left(\frac{1}{S} \oint_s \vec{H} \cdot d\vec{s} \right)$$ Rotation (Curl)

Here we have introduced the magnetic field strength $\vec{H}$ instead of the force $\vec{F}$ whereby $\vec{H}$ is not necessarily curl free. s is the closed circulation path and S is the surface limited by s. Despite the possible confusion, the surface area is not depicted by the letter A as common in mathematics, because A or $\vec{A}$ denote the vector potential.

The following sentences of the field theory are given here without proof, full particulars can be found in the cited mathematics literature:

- A solenoidal vector field can always be depicted by the rotation of a vector potential.
- A curl free vector field can always be represented as the gradient of a scalar potential.

The relationships collected in **Fig. 4.33** can be understood most simply if one starts with the flux density. Magnetic fluxes are solenoidal because no magnetic monopole exists and, thus, the divergence is zero. This statement is depicted as Maxwell's Third Law, but sometimes also as Maxwell's Fourth Law – physicists do not have a common denomination: Since $\vec{B}$ is solenoidal one can always specify a corresponding vector potential $\vec{A}$. Using the permeability μ, one gets from the flux density $\vec{B}$ to the field strength, the line integral of which will yield the magnetomotive force V, which is coupled to the flux Φ through the magnetic conductivity Λ. The flux is the surface integral over the flux density $\vec{B}$. If one integrates the field strength over a closed loop s one will get the integrated magnetic field Θ. This equals the current I enclosed by s, which can be depicted as surface integral of the current density $\vec{J}$ over a surface S limited by s. The relationship between the field strength $\vec{H}$ and the current density $\vec{J}$ is described by Maxwell's First Law: its integral form equates the integrated magnetic field and the enclosed current (Ampere's Circuital Law). Its differential form connects the current density and the rotation of the field strength. In current-free areas the rotation of the field strength is zero and, thus, the field strength is curl-free and can, consequently, be interpreted as the gradient field of the scalar potential ψ.

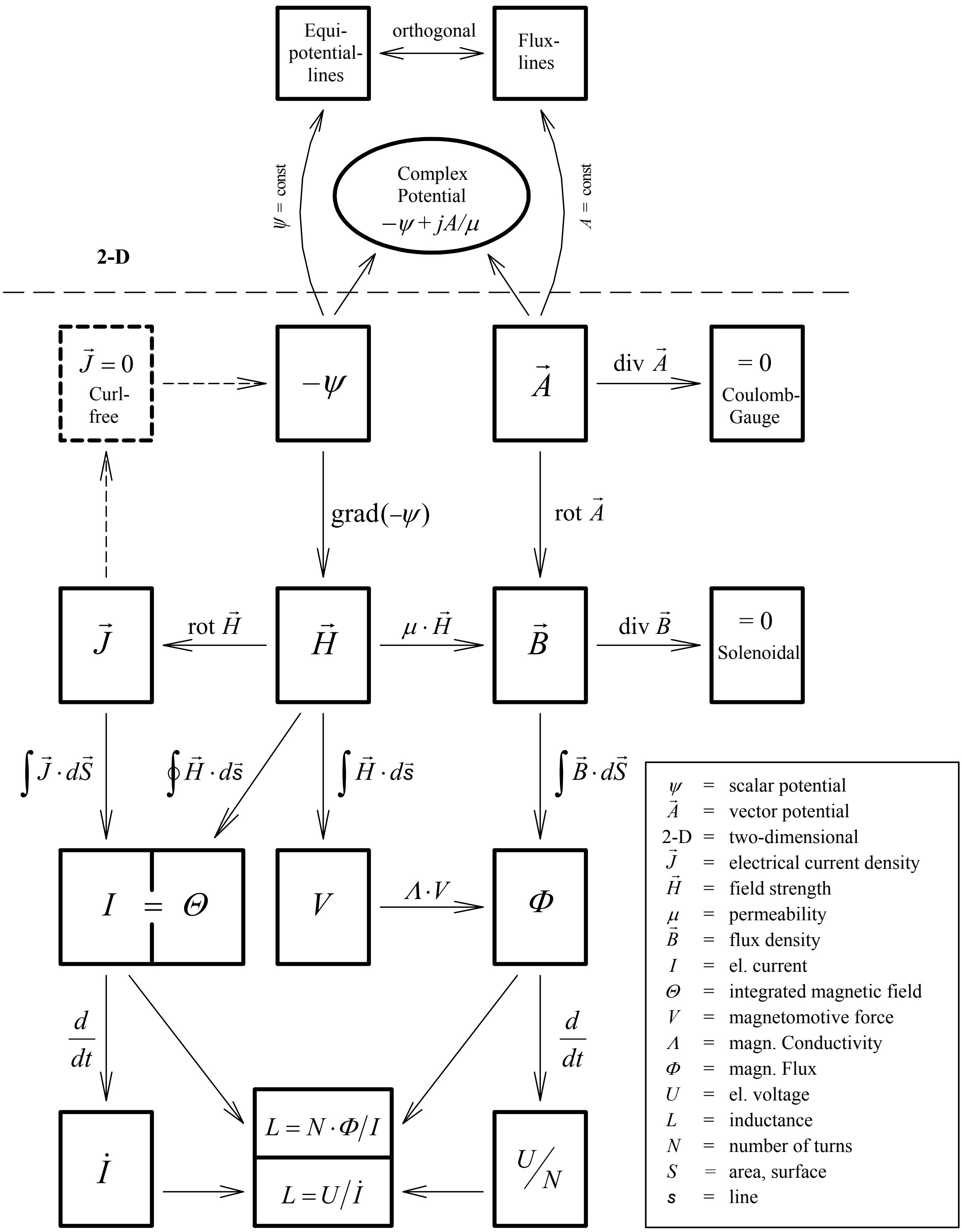


Fig. 4.33: Formal relationships between the magnetic field values. The complex potential is only defined for 2-dimensional fields. The scalar potential is only defined for current-free regions. This representation does not include wave propagation processes.

Under the assumption that μ is constant, i.e. that it is not position-dependent, $\vec{H}$ as well as $\vec{B}$ are curl-free and solenoidal. For the scalar potential this yields

$$\Delta\psi = \operatorname{div}\big(\operatorname{grad}(\psi)\big) = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = 0 \qquad \text{Laplace Equation}$$

This is the (homogenous) Laplace-Equation. If the Laplace-Operator Δ is applied to the vector potential instead of the scalar potential, the result is also zero:

$$\Delta\vec{A} = \operatorname{grad}\big(\operatorname{div}(\vec{A})\big) - \operatorname{rot}\big(\operatorname{rot}(\vec{A})\big) = 0\,; \qquad \operatorname{div}(\vec{A}) \overset{!}{=} 0 \qquad \text{Coulomb Gauge}$$

Basically, an integration from $\vec{B}$ to $\vec{A}$ offers an optional integration constant; it is chosen such that the divergence of $\vec{A}$ is zero (Coulomb-Gauge of the vector potential). Thus, the above given scalar Laplace-Equations are valid for every vector component of $\vec{A}$:

$$\Delta A_x = 0, \qquad \Delta A_y = 0, \qquad \Delta A_z = 0. \qquad \text{Vector Laplace Equation}$$

The Laplace-Equation is a very general linear homogenous differential equation. It can be applied to describe a solenoidal and curl-free field with only a single equation. However, within current carrying regions, the magnetic field is only solenoidal and not curl-free. Consequently, a scalar potential does not exist. For this application one employs the (inhomogeneous) Poisson-Equation:

$$\Delta\vec{A} = -\mu\cdot\vec{J} \;\longrightarrow\; \operatorname{rot}\big(\operatorname{rot}(\vec{A})\big) = \mu\cdot\vec{J}\,. \qquad \text{Poisson Equation}$$

The Laplace and Poisson equations assume a locally constant μ. However, in ferromagnetic materials the permeability is dependent on the field strength which is dependent on the position, yielding a location dependent permeability – the Laplace and Poisson equations are, consequently, not valid without restrictions.

A **complex potential** F can be defined in the **two-dimensional parallel plane** field. The formula symbols f or F are used in the mathematical literature; both characters are problematic because, in electrical engineering, f stands for frequency and F for force. In order to distinguish, F will subsequently be used. The components of the complex potential are **differentiable** complex functions, which are also called holomorphic, analytical or **regular**. Regular potential functions are invariant with respect to conformal mapping and, yielding simpler options for characterization and computation. The air flow around a complex airfoil can be projected onto two simple circles; the magnetic field around a cylinder can be mapped as a superposition of a parallel and a dipole field. Here, the orthogonality between equipotential and flux lines is conserved, because the conformal projection is isogonal. However, one may soon notice that the magnetic fields of pickups cannot be described sufficiently in two dimensions. The theory of the complex potential is only useful as a starting point to explain the basic relationships. The two cylinder axes are orthogonal for a cylinder magnet and a string; this situation cannot be described by parallel plane or by rotational symmetry. Rather, a 3-dimensional coordinate system is necessary – and the complex potential is not defined within this boundary condition.

The point variable of the complex potential is $z = x + jy$. Here x and y are the abscissa and the ordinate of a two-dimensional coordinate system, respectively. The scalar potential ψ is a regular potential function of z, for which the differentiability and the regularity are given by the applicability of the Laplace differential equation. The scalar potential is then considered to be the real part of the regular complex function F or, in other words, the real part function ψ is supplemented by an imaginary part to become an **analytical function**. This imaginary part is defined by ψ in a unique way, because F should not become any complex function but a regular (=analytical) one. The Cauchy/Riemann differential equations (**C/R-DGL**) are valid for regular functions and, as a result, an imaginary part can be derived for every real part. As for every integration, an additive constant can be freely chosen. It is determined by the Coulomb Gauge.

The definition of the complex potential is, however, yet not entirely **unique**: One could consider ψ also as imaginary part to which a real part is supplemented and also the sign can be arbitrarily assigned. In general notation the complex potential is:

$$F(z) = u(z) + jv(z) \qquad \text{Complex potential}$$

The C/R-DGL have to be valid because F should be a regular function in the region under consideration:

$$\partial u/\partial x = \partial v/\partial y, \qquad \partial v/\partial x = -\partial u/\partial y \qquad \text{Cauchy/Riemann}$$

Mathematics interprets u to be the scalar potential of a curl-free vector field, which can be depicted as the gradient of u. The gradient is a vector which points in the direction of the largest field increase. However, **physics** defines the magnetic scalar potential ψ (also the electrical scalar potential φ) as a vector pointing in the largest field decrease. It is, therefore, obvious to assign a minus-sign to the real part of F:

$$F(z) = -\psi(z) + jv(z) \qquad \text{Sign facultative}$$

The gradient of the real part of F is the field strength vector, the components of which can be translated into the imaginary part of the complex potential with the help of C/R-DGL:

$$\vec{H} = -\mathrm{grad}\,\psi = \begin{pmatrix} -\partial\psi/\partial x \\ -\partial\psi/\partial y \end{pmatrix} = \begin{pmatrix} \partial v/\partial y \\ -\partial v/\partial x \end{pmatrix} \qquad \psi = \text{scalar potential}$$

A corresponding relationship can be derived for the vector potential, whose rotation yields the flux density. In two dimensions (for which the current description is valid) the **vector potential** has only one component $A = A_z$. The index z here refers to the third Cartesian coordinate of the x-y-z-system; it should not be mixed up with the complex space coordinate $z = x + jy$ of the two-dimensional field!

$$\vec{H} = \frac{1}{\mu}\vec{B} = \frac{1}{\mu}\mathrm{rot}\vec{A} = \frac{1}{\mu}\begin{pmatrix} \partial A/\partial y \\ -\partial A/\partial x \end{pmatrix}, \quad v = A/\mu, \quad \mu \neq \mu(z). \qquad \vec{A} = \text{vector potential}$$

It follows for the **complex potential** of the magnetic field:

$$F(z) = -\psi(z) + jA(z)/\mu \qquad \text{Complex potential}$$

The complex potential is a combination of the scalar potential ψ and the vector potential $\vec{A}$, which are, in turn, functions of the field strength. The rotation of the field strength is zero in the **curl-free** magnetic field:

$$\mathrm{rot}\vec{H} = 0 \quad \longrightarrow \quad \partial H_y / \partial x - \partial H_x / \partial y = 0 \quad \longrightarrow \quad \partial H_y / \partial x = \partial H_x / \partial y$$

This is the so-called *integration condition* of a plane vector field, a necessary and sufficient condition for the independence of the line integral on the integration path and for the complete differential $d\psi$:

$$-d\psi = -\frac{\partial \psi}{\partial x}dx - \frac{\partial \psi}{\partial y}dy = H_x \cdot dx + H_y \cdot dy \qquad \text{Complete differential}$$

For $d\psi = 0$ one obtains curves of constant scalar potential ψ = const., the so-called equipotential lines. The slope dy/dx of these lines correlates with $-H_x/H_y$, i.e. the equipotential lines are normal to the direction of the field strength vector.

In a **solenoidal** magnetic field the divergence of the flux density is zero and, hence, the divergence of the field strength is zero for a location independent μ:

$$\mathrm{div}\vec{H} = 0 \quad \longrightarrow \quad \partial H_x / \partial x + \partial H_y / \partial y = 0 \qquad \text{for } \mu = const\text{, i.e. } \mu \neq \mu(z)$$

The complete differential dA of the vector potential A leads to:

$$dA = \frac{\partial A}{\partial x}dx + \frac{\partial A}{\partial y}dy = -\mu H_y \cdot dx + \mu H_x \cdot dy \qquad \text{with } \mu\vec{H} = \mathrm{rot}\vec{A}$$

One gets curves of equal vector potential (A = const.) for $dA = 0$, whose slope dy/dx corresponds to the slope of the field strength vector: $dy/dx = H_y/H_x$.

Hence, curves of equal vector potential form the directional field of the field lines, with curves of equal scalar potential (equipotential lines) being normal to them.

Since the complex potential F is defined as a regular (analytical) function, every regular projection of F must result in a regular function of z. The complex derivative d/dz is such a regular projection (satisfying C/R-DGL); if applied on the complex potential this yields:

$$dF(z)/dz = \frac{\partial}{\partial x}\mathrm{Re}\{F\} - j\frac{\partial}{\partial y}\mathrm{Re}\{F\} = -\frac{\partial \psi}{\partial x} + j\frac{\partial \psi}{\partial y} = H_x - jH_y = H^*$$

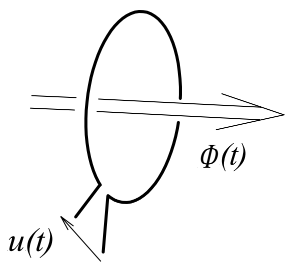
The derivative of the complex potential corresponds to the conjugated complex field strength, whose x and y components are interpreted as the real and imaginary parts. H^* is also a regular function of z. The complex integral over the conjugated field strength is the complex potential. The additive constants for ψ and A are arbitrary for this integration; they define the origin of the scalar and vector potentials.

4.10 Magnetodynamics

Dynamics is derived from the Greek word dynamos = force, which is why dictionaries like to explain this word as "tenet of the force". However, magneto-*dynamics* does not primarily deal with forces but, contrary to magneto-*static*s, it deals with systems whose signals or system variables are experiencing variations. In this case it can be changes of location or movement as well as temporal changes of fields in a stationary system. Ultimately, forces may also be involved, but they are not in the foreground.

4.10.1 Magnetic Voltage Induction

An electric voltage is induced in a conductor loop, which correlates with the temporal change of the percolating magnetic flux. This is the basic principle of voltage generation in a magnetic pickup. The flux is the product (in general the integral) of the flux density and the area. A change can therefore arise as a change of the flux density and/or area. In a pickup the conductor is formed by many loops of coiled copper wire. For high quality manufacturing the single loops are glued together in such a way that the coil area remains constant so that the only changes are in the flux density. The source and origin of the flux change is the vibrating string, the effect of which can be described in two different ways: changes of location of the string alter the magnetic resistance in the magnetic circuit, resulting in flux density changes (magnetic transducer). Alternatively, the string can be considered as being magnetized by the pickup; movements of the string are, therefore, relative movements between the string magnet and the coil (dynamic transducer).



$$u(t) = N \cdot d\Phi / dt$$ Induction Law

Fig. 4.34: If the magnetic flux *Φ(t)* increases with time, a positive voltage *u(t)* will be induced.

In **Fig. 4.34** a wire loop is depicted which is penetrated by a flux that increases with time. The voltage *u(t)* shown forms as a consequence of the change in flux. Sometimes, the induction law is also written with a **minus sign**, depending on the reversed definition of the arrow.

For a **guitar pickup** there is not only *one* loop, rather the wire is wound to a **coil** with N = 5000 – 10000 turns. If one calculates from $\tilde{u} = 1\text{V}$ back to the magnetic parameters, one will get a change in flux density of 1 mT (peak value $\hat{B}$) for f = 2 kHz, N = 6200 turns and a magnet area of 18 mm^2. The relative change in the flux density caused by the vibrating string is, thus, only approximately 1% compared to the static flux density of the permanent magnet (approx. 100 mT).

More accurate considerations show that it is difficult to compute the induced voltage in the pickup coil. Not only does the string change its position in space, it also warps (bends) while it vibrates. For the calculation of the flux change one would have to perform a three-dimensional field calculation, including the non-linear *B/H* behavior of magnet and string. Further, one has to take into account that not every single loop of the coil is penetrated by the same magnetic flux: the field that is generated by the string diverges and loops that are located closer to the string will experience a larger flux than those further away. Despite these difficulties one can, with restrictions and approximations, realize a reasonable agreement between theory and observation.

Two questions are particularly pertinent when using the **Induction Law**: How big is the induced voltage and which type of curve is generated? As the movements of the strings are non-linear projections of the change of flux, sine-like vibrations will not yield sine-like voltages. Chapter 5.8 will address pickup-distortion in more detail. We will only investigate the small signal behavior here. The flux change will, therefore, be assumed to have a single frequency, be sine-like and be described by the frequency f and by the effective value of the flux density B. The time differential, thus, simplifies to a product:

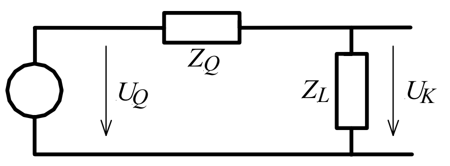
$$u = 2\pi f \cdot N \cdot B \cdot S$$

Induction Law for a sine-like flux density

where N is the number of turns, B is the effective flux density, and S is the area of the coil; u is the effective value of the induced voltage. Since all turns of a coil are wound in the same direction, the voltages induced in every single loop are adding up to the total coil voltage (typically some 100 mV, max. approx. 5 V).

4.10.2 Self-Induction, Inductance

The voltage that is induced in the coil should be interpreted as a **source voltage**, not as a terminal voltage. It evolves, so to say, in the interior of the pickup, just as if an alternate voltage source is built in there. The voltage measured at the clamps (pickup cable) only equals the source voltage for the case without **load**, i.e. in **open source** mode. Once a load is applied, "the terminal voltage breaks down", i.e. it becomes smaller than the source voltage. This behavior can also be observed in the lighting main: If one switches on a 2 kW furnace, the light dims. The reason for the voltage decrease is the voltage drop across the internal resistance, which results in a **voltage divider** (**Fig. 4.35**):



$$U_K = U_Q \cdot Z_L / (Z_Q + Z_L)$$

Fig. 4.35: Voltage divider between load impedance Z_L and source impedance Z_0.

For the general case the load and source resistors are frequency dependent, which is why one speaks of load impedance Z_L and source impedance Z_Q. The induced voltage is U_Q and at the terminals U_K builds up. Both voltages are only identical for an infinite load impedance (open circuit).

Figure 4.35 depicts a circuit mesh in which a current can flow. The required **energy** is delivered by the ideal voltage source, which is drawn as a circle. Of course, the energy cannot emerge from nothing. If the pickup should deliver energy, it must be provided with energy. And it is: energetically, the source of the pickup terminal voltage is the vibrating string. Its vibration energy is diminished a little bit by the pickup (and the connected resistances), where electrical energy is transformed into heat. In other words, the electrical resistances in Fig. 4.35 retroact on the string and change its vibration so that the source voltage is load dependent. However, as this effect is only very small, it is neglected here. The source voltage U_Q is considered to be imprinted, it equals the induction voltage defined in the preceding chapter.

The **load impedance** is composed of the coil capacity (see pickup parameters), the guitar electronics (volume and tone potentiometers), and of the guitar cable and amplifier. Simplifying, this can be described by a parallel circuit of 300 – 1000 pF and 100 – 350 kΩ. Further, the **source impedance** is formed by the coil. Here, one first deals with approximately 1 km of thin enameled copper wire with a DC resistance of approx. 5 -15 kΩ. Another effect has to be considered for AC, and only AC flows as consequence of the induced alternating voltage: as already shown in chapter 4.1 every alternating current produces a magnetic field in its vicinity: an *alternating* current produces an alternating field, i.e. a field with alternating polarity. This field also percolates through the pickup coil and induces a voltage. As this field is produced by the pickup *itself* (in contrary to the field produced by the string), the voltage which is built up as a result is called the **self-induction voltage**. The self-induction voltage superimposes itself inversely phased on the voltage originating from the string vibration and weakens it (Lenz's rule). It is obvious that this voltage superposition cannot be in phase because otherwise the current would increase with increasing voltage, yielding voltage increase, yielding current increase … and the system would become unstable. In order to take into account the self-induction induced voltage decrease, one could include an additional regulated voltage source in Fig. 4.35, whose source voltage is dependent on the current flowing in the mesh. However, it is common to instead draw in a component, whose voltage drop is equal to the self-induction voltage. In the circuit it is symbolized either by a black, filled rectangle, or by a symbolic representation of the wire coils (**Fig. 4.36**); this component is an **inductive two-terminal device** (= **inductor**), the unit symbol for an inductor is L.

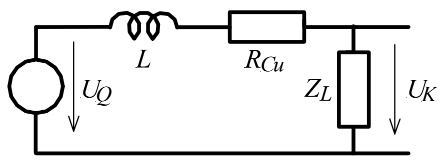


Fig. 4.36: Equivalent circuit of the coil with inductor L and resistance of the copper wire R_{Cu}.

In a Gedankenexperiment (thought experiment) we will now allow a constant current to flow through the circuit depicted in Fig. 4.36. This constant current can, however, not be generated by induction as U_Q, but it could be fed in at the clamps depicted on the right. As consequence of this constant current, a constant magnetic field would be generated, whose flux differential (with time) yields the induced voltage. The differential of a constant is, of course, zero – consequently, the constant voltage drop at an ideal inductor must be zero too.

However, if an AC current flows, an induced voltage develops, the quantity of which is depending on the current change:

$$u(t) = L \cdot di(t)/dt; \qquad \underline{U} = j\omega L \cdot \underline{I}$$ Two terminal equations

A voltage drop *u(t)*, which is proportional to *L* and to the current change with time, will be generated in an inductor *L* in which a current *i(t)* flows. A representation with rotating complex pointers is convenient for sinusoidal oscillations. Using this, the time differential will transform into the factor $j\omega$. The imaginary unit *j* will yield a rotation (phase shift between voltage and current) of 90°, $\omega = 2\pi f$ is the angular frequency. The derivative *d/dt* and multiplication with $j\omega$ are linear operations; they do not destroy the proportionality between current and voltage. For direct current the proportionality coefficient between *U* and *I* is called the **resistance** ($U = RI$), for alternating currents it is called **impedance** instead. The impedance of the inductor is $j\omega L$; for direct current it is zero, with increasing frequency it increases in proportion to the frequency. The impedance of an inductor is a positive imaginary quantity (precisely: not negative), one could also say: the resistance of an inductor is positively imaginary.

The **unit** of the inductance is **Henry**: 1H = 1Vs/A.

The letter H should not be mixed up with the formula symbol *H*, which stands for the magnetic field strength! The quantity of inductance of *L* can be deduced from the geometry of the coiled wires. Typical values for a guitar pickup are *L* = 2 - 10 H.

Equations for the **calculation** of simple coil inductivities are quoted in every book on magneto-dynamics. Simple formulas are obtained for the toroidal coil and the long cylinder coil. However, for the magnetic pickup the conditions are more complicated: the magnetic field generated by the vibrating string is inhomogeneous, i.e. dependent on the position. Thus, every turn of the pickup coil will be penetrated by different magnetic fluxes and an analogy, as in Fig 4.35, with one single voltage source and one single inductor is not possible in the first instance. This effect is substantial, it cannot simply be ignored: for a Stratocaster pickup the magnetic fluxes in a turn near the string and in a turn away from the string differ by a factor of 10 (chapter 5.4.3). For the calculation of the quantity of the induced voltage one has to perform appropriate suitable averaging. In addition, one has to consider that the magnetic field will be focused (enhanced) by ferromagnetic materials. The Alnico-magnets placed inside the coil are ferromagnetic and focus the magnetic flux, which yields a higher inductance in comparison to a coil which is free of magnetic fields (see chapter 4.10.3).

Since the alternating magnetic flux has its maximum strength in close vicinity to the string, for efficient conversion ('loud pickup') it is recommended to locate the coil as near as possible to the string. One may find Fender pickup designs with magnets ending right at the edge of the flange facing the string; they are 'loud'. However, there are also pickups (the ones with '*staggered magnets*') with magnets extending up to 4 mm; they produce (as a rough approximation) only half of the voltage. Naturally, this rule of thumb presumes that all other parameters remain constant. In particular, the contour of the coil can deliver an additional degree of freedom: of equal height or conically wound.

4.10.3 Permeability

In chapter 4.3 the permeability was defined as the *specific magnetic conductivity*. It is a material property like, e.g. the electrical conductivity in a current circuit. Air has a relatively poor magnetic conductivity (μ_0 = 1,257 μH/m) and the magnetic resistance of air is relatively high. The permeability of magnetic materials is often depicted in relation to the permeability of air♣ as the **relative material permeability**: $\mu = \mu_r \cdot \mu_0$. Here, μ_r is the relative permeability, also called **permeability number**.

The permeability number of air equals one, to very good approximation The difference to vacuum is not significant. In contrast to air, in which the relationship between the magnetic field strength H and the magnetic flux density B in air is proportional: $B = \mu \cdot H$, in ferromagnetic materials (Alnico, steel, iron, nickel) the permeability number is larger than one and dependent on the magnetic field strength. The literature for iron claims maximum permeability numbers of approx. 5000, but also up to 250000, if one deals with purest iron heat treated in hydrogen. If one would multiply the field strength in the close vicinity of a pickup of 40 kA/m with $5000 \cdot \mu_0$ this would yield, in purely mathematical terms, a flux density of 251T. But this value has of no practical meaning because, on the one hand, generally the magnetic field strength changes when materials with magnetic conductivity are inserted into the field, on the other hand the saturation value for the magnetic flux density of iron is approx. 1 T. **Saturation** means that the field enhancement in iron reaches a limit that cannot be exceeded. The origin of a magnetic field is supposed to be a moving charge and, hence, an electric current. In all magnetically non-neutral materials internal circular currents (elemental currents) align themselves with respect to external magnetic fields and form an internal magnetic field, which is superimposed to the external one. The flux density B generated in the iron can be considered as being the sum of an externally generated component $B_0 = H \cdot \mu_0$ and an internal field enhancing **polarization** J. In other words, the component B_0 forms principally and everywhere in the magnetic field. In magnetic materials the polarization J is added to it.

$$B = B_0 + J\,; \qquad J = \chi_m \cdot \mu_0 \cdot H\,; \qquad \chi_m = \mu_r - 1 = \text{relative susceptibility}$$

The relationship between B_0 and H is proportional; it is not subjected to saturation. However, the polarization depends decreasingly on H and tends to a non-exceedable limit = saturation polarization J_{sat} (**Fig. 4.37**).

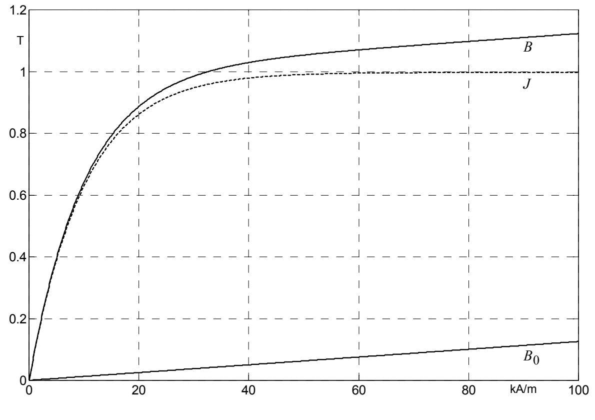


Fig. 4.37: Relationship between field strength H, flux density B and polarization J. In this example the saturation polarization is 1 T.

♣ Actually, vacuum is the reference: $\mu_{\text{air}} = 1.0000004 \times \mu_{\text{vacuum}}$.

In the pickup an electrical voltage can only be generated (induced) if the magnetic flux density B changes as a function of time t and, as B depends on H through μ, the magnetic field strength H also changes. The calculation of such field changes is highly complicated and only possible as a rough approximation. The string and the pickup-magnet are ferromagnetic in different ways and, in addition, the screws, pole pieces and plates may possibly influence the field. In every differential metal volume a different B and, hence, a different μ can dominate. Adding to the difficulty, small changes of the field ΔH lead to small changes in the flux density ΔB which cannot be deduced from the slope of the magnetization curves (hysteresis loops). In **Fig.4.38** in the right picture we have depicted a section of the magnetization curve. If the field strength increases from the working point H_A by a small amount ΔH, the corresponding flux density will adjust to $B_A + \Delta B$, which is not located on the large continuous curve; this one can only be traversed in the direction of the arrow. The quotient from $\Delta B / \Delta H = \mu_{rev}$ is called the **reversible permeability**. It is smaller than the **differential permeability**, which can be viewed the as differential quotient or slope of the hysteresis (dashed line in the figure). According to [7] the reversible permeability not only depends on flux density B, according to Gans [Phys. Z., 12/1911], but also on on the polarization J. It also has – and this complicates the numerical calculations – a tensor character for isotropic media!

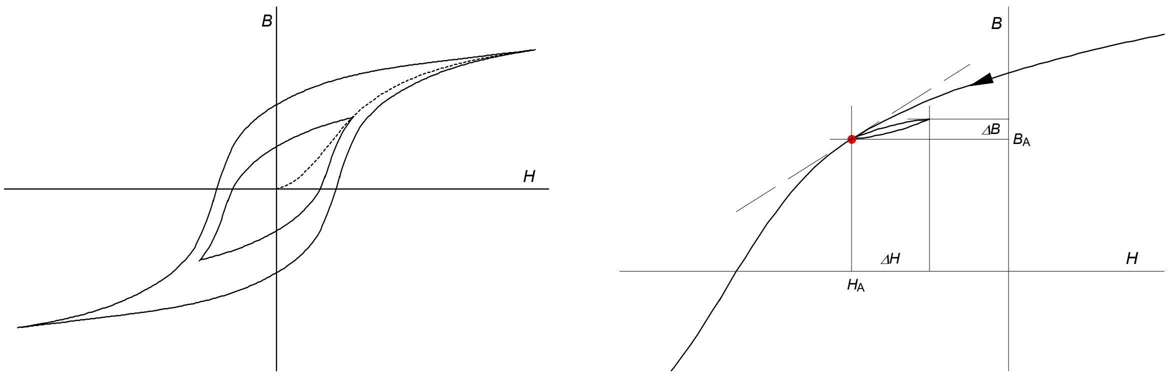


Fig. 4.38: Difference between differential and reversible permeability. ΔH and ΔB represent small changes; in the picture they are considerably exaggerated (cf. Fig. 4.6).

The reversible permeability μ_{rev} is maximum for $B = 0$ and decreases monotonically towards μ_0 with increasing value of the flux density. Consequently, ferromagnetics conduct alternating magnetic fields much more poorly the higher the static percolating magnetic flux is. Thus, it is incorrect to simply assign a better magnetic conductivity to a **steel-string** because steel is depicted as ferromagnetic material in tables ($\mu >> 1$). Only steel strings without static pre-magnetization may exhibit permeability numbers which exceed 50. However, as soon as a string is in the vicinity of a pickup magnet, a considerable static magnetic flux will flow through it, and the reversible permeability will drop to values only slightly higher than that of air (**Fig. 4.39**)

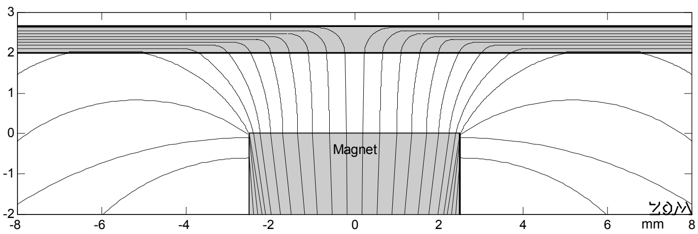


Fig. 4.39: Approximate progression of the static magnetic flux for magnet, air gap and string (→ Fig. 5.4.8).

In **Fig. 4.39** one can recognize, how the static magnetic flux density (depicted as line density) increases within the string along the string axis. Directly above the magnet axis the axial string flux density is zero, but already after approx. 6 mm a maximum is reached which practically means full saturation (1.7 Tesla, steel string with Alnico-5-magnet, chapter 5.4.2). If now the string loses its good conductivity for alternating magnetic fields already after some millimeters, they will have no reason to follow the string and, consequently, leave it. Thus, vibrations of the string only influence the magnetic field in a relatively small volume and the **magnetic window** (the magnetic aperture) is relatively short (chapter 5.4.4). The closer the magnet is located to the string and the stronger it is, the shorter is the magnetic aperture (the string sampling is more selective) and the corresponding damping of the treble frequencies is lower.

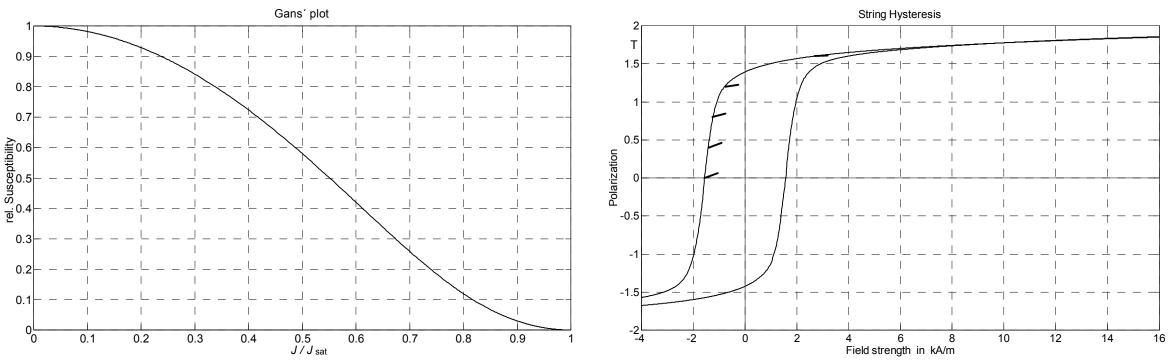


Fig. 4.40: Gans curve for the reversible susceptibility (left) and string-hysteresis (right).

In **Fig. 4.40** we have depicted the **"Gans curve"** in the left picture. It describes the relationship between the normalized polarization up to the saturation polarization J_{sat} and the normalized reversible susceptibility up to the starting susceptibility χ_A. Their formula [21] reads, in parameter representation:

$$\frac{\chi_{rev}}{\chi_A} = 3 \cdot \left(\frac{1}{x^2} - \frac{1}{\sinh^2(x)} \right); \qquad \frac{J}{J_{sat}} = \coth(x) - \frac{1}{x}; \qquad \text{Gans curve}$$

In this parameter representation x stands for the parameter (0 … 100), J for the polarization and χ_{rev} for the reversible magnetic susceptibility. The starting susceptibility acts at the beginning, i.e. at the origin of the new (first) curve. In the right picture of Fig. 4.40 the hysteresis curve of a steel string is shown, together with little lines indicating the slope of small signal changes. **Fig. 4.41** shows the measured results of a typical steel string.

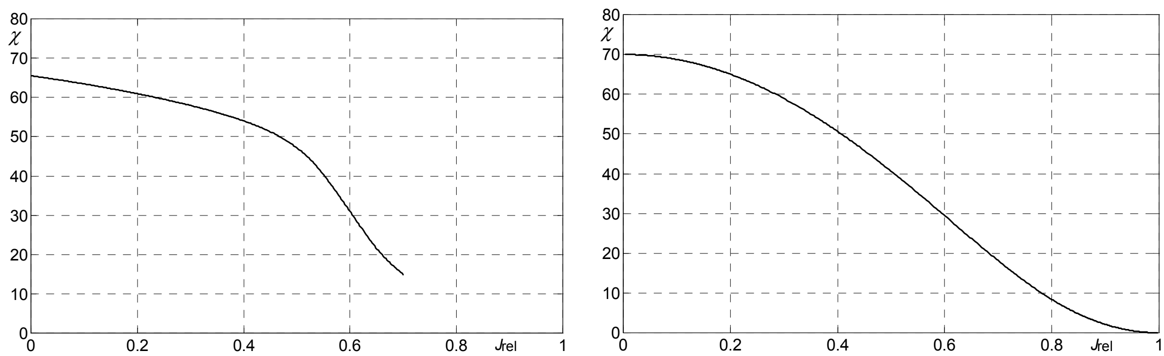


Abb. 4.41: Measured reversible susceptibility (left) and theoretical simplification of the Gans curve (right).

4.10.4 Magnetic Losses, magnetic skin-effect

The field enhancing effect of ferromagnetic materials is caused, on the one side by the internal shift of domain walls (Bloch walls, chapter 4.4.1), and on the other side by initially randomly oriented elemental magnets that are turned into a common direction by the external field. A small part of the energy that is necessary for shifting and/or rotating is irreversibly transferred into heat. The thermal energy that is produced by a kind of micro-friction is “lost” from the electromagnetic field and this is the reason why one talks about **loss of electromagnetic field energy**, or in short about magnetic losses; the designation **iron losses** is also common. Losses will decrease the voltage generated in the pickup – an effect which may mainly affect higher frequencies as brilliance loss.

The two most important loss mechanisms are eddy current losses and hysteresis losses. The field-energy per volume w can be derived from the relationship between the magnetic field strength H and magnetic flux density B, as given by the hysteresis curve:

$$w = \int_{B_1}^{B_2} H dB$$ Volume-specific magnetic field energy

If the hysteresis curve is a (curved) line, the magnetic energy would be increased by elevating the flux density from B_1 to B_2 and likewise would be diminished by the same value by decreasing from B_2 to B_1 – the process would be reversible. However, as each hysteresis loop consists of two different branches, a complete circuit leading back to the origin does not yield $w = 0$ but an energy density which is proportional to the enclosed area and which represents a measure for the energy loss. For guitar strings, the specific **energy loss** for a *boundary loop* circuit is about 10 µWs/mm³. If one multiplies this value with a 2 cm long 0.7 mm string one ends up with an energy loss of approximately 77 µWs for the total hysteresis circuit. However, the working point of a vibrating string does not follow the boundary hysteresis (from negative saturation to positive saturation and back) but only a small fraction of it. The fraction of it heavily depends on the distance of the string to the magnet and on the amplitude of the string deflection. The steady flux is also high in the regions of high alternating flux and – conservatively estimated – the alternating flux may reach about one tenth of the steady flux. In addition, if one considers that the small signal changes yield relatively small areas, lancet-shaped hysteresis loops (also called **Rayleigh-Loops**), it becomes clear, that the energy losses caused by the string are of only marginal significance. As an order of magnitude one can estimate 10 mWs for the string energy and 1 µWs for the iron losses per cycle. If the string vibrates with 150 Hz with this assumption it will lose 1.5% of its vibration energy, which would be negligible. A more precise computation of the iron losses would be laborious, because one has to deal with a three-dimensional inhomogeneous field, for which material tensor parameters would have to be known. In addition, measurements are difficult because one has to discriminate from other damping mechanisms. But, even for the case that the above approximation would be unrealistic and the string energy loss per second would be 26% instead of 1.5% this would be equal to a level decrease of 1 dB/s – insignificant against other damping mechanisms. The bottom line of these approximations is, therefore, (without proof): the **hysteresis losses** (magnetization-change losses) **emerging within a string are negligible**.

Other than in the string, hysteresis losses are also possible in the magnet or in nearby ferromagnetics. Although these losses are not generated within the string, nevertheless the energy necessary for changing the magnetization of these ferromagnetics has to be delivered by the vibrating string. The magnetic volume affected by relevant alternating fluxes for single coils is larger, by more than one order of magnitude, compared to the above considered volume of the string. However, the relative change in flux density inside the magnet is also one order of magnitude smaller than inside the string so, on the whole, again an effect of marginal importance. As long as one does not move a very strong magnet close to the string (which is in contradiction to a large string displacement) **the conclusion is: hysteresis losses are negligible**. This statement is indeed speculative but is supported by measurements which yield, without doubt, that string vibrations are damped more intensely by the fretting hand of the guitarist than by the magnetic field of the pickup (chapter 4.11).

A second source for losses are **eddy current losses**. The induction law discussed in chapter 4.10.1 generates a voltage and a current in the pickup coil but also in every conductor that is in close vicinity to the pickup. As metals represent electrical resistances, an effective electrical energy or thermal energy is produced which weakens the magnetic field or the string vibration. The electrical voltage that causes the eddy current is dependent on the *change* of the magnetic field and, therefore, eddy currents do not play any role at low frequencies. With increasing frequency they become more and more important, however, the **skin effect** has also to be taken into account as reverse effect (chapter 5.9.2.2): the magnetic counter field induced by the current flow forces the current more and more into the boundary regions and consequently increases the eddy current resistance (chapter 3.3.2)

Eddy current losses cannot generally be neglected, but indeed deteriorate the treble reproduction of every pickup; not only marginal but possibly by 5 dB and more, if thick low-ohmic metal plates are employed. One could interpret this fact as a sound characteristic which is deliberately chosen by the developer, but one should take into account that very dominant treble can be reduced easily by a potentiometer in parallel to the pickup, which is not possible the other way round. A pickup with less eddy currents may sound brilliant as well as dull; a pickup damped by eddy currents may only sound dull[♣]. Pickups that exhibit small eddy current losses are the ones with 6 Alnico magnets as sole metal pieces (USA-type Stratocaster). Soft-magnet pole-pieces with underlying bar magnets increase the eddy currents, as do tin covers. If one wishes to have a shielding case with small eddy current losses, thin-walled German silver cases are recommended. One pickup that sounds brilliant, despite having a metal case, is the Gretsch humbucker.

Eddy currents are not only present in magnets, pole-pieces and shielding cases but are also possibly in metal support plates and shielding foils. When replacing a plastic by an aluminum pick-guard one experiences a small treble loss. However, the loss can mostly be avoided by a small slit, which suppresses the circling eddy currents.

[♣] In general, sound filters built into guitar amplifiers cannot compensate for eddy current losses.

In order to obtain quantitative data for **eddy current losses**, a thin walled measuring coil was fabricated, into which cylinder-shaped ferromagnetics (∅ = 5mm) could be inserted. The 14 mm wide coil form was wound with 5500 turns of an 80-μ-CuL enameled copper wire (**Fig. 4.42**). In this representation the logarithmic impedance unit is depicted over the logarithmic frequency – unusual but convenient. 0 dB = 1 kΩ.

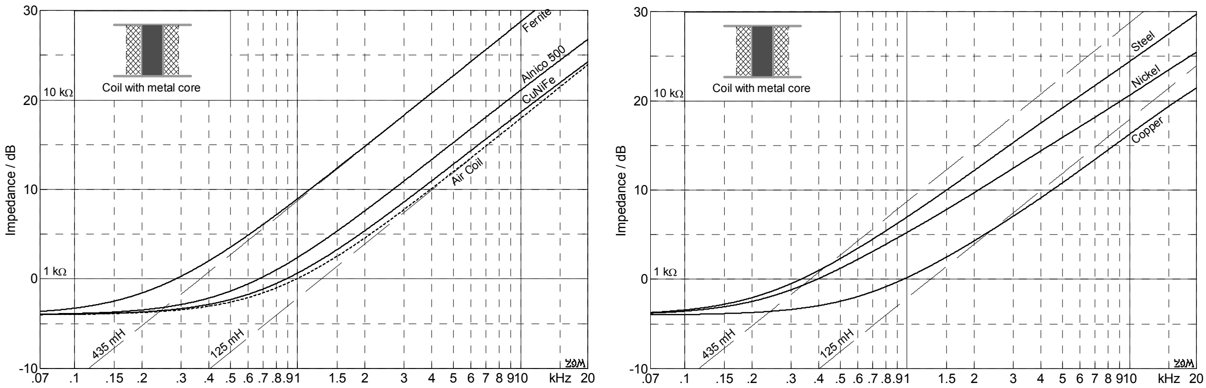


Fig. 4.42: Logarithmic impedance of a measuring coil ($N = 5500$); different core materials.

The wire resistance (630 Ω) is measured without a core ("air coil") at low frequencies and at high frequencies the impedance increase proportional to the frequency, the inductance (125 mH). The two terminal device is, thus, perfectly described by an *RL*-series connection in this frequency range. Insertion of an **Alnico**-500-magnet (5x14 mm) increases the inductance by 46%, insertion of a respective **ferrite** cylinder increases the inductance by a factor of 3.5. In both cases a frequency-proportional inductance increase happens at high frequencies, so that only one resistance is necessary in the equivalent circuit: the **wire resistance**♥. The inductance increase, however, does not mean that the relative permittivity of ferrite is only 3.5 (or for Alnico is only 1.46). These materials cover only a part of the field space, their effectiveness is, thus, substantially diminished. As an analogy one might think of two resistors in series, e.g. 1000 Ω and 10 Ω. The total resistance in this example is 1010 Ω. It decreases to 1001 Ω if the second resistor has only 1 Ω. At a 1V-source a current of approx. 1 mA will flow even if one will decrease the second resistor even further. This is similar for the magnetic circuit: the magnetomotive force is dominated by the low-conductive air field. With a little peculiarity: A change in the magnetic resistance of the core will also affect the shape of the field lines and, thus, the resistance of the air field.

The reason, why the impedance of Alnico and ferrite can be represented by an ordinary *RL*-two terminal device, is quite simple: in addition to the wire resistance no additional loss resistance has to be taken into account: eddy currents do not yet play a role♣. **Ferrites** are sintered out of oxide powder; they have a high electric resistance that prevents eddy currents. **Alnico**-alloys are, in comparison to ferrites, already quite good conductors. The fact, that they exhibit nearly no eddy current losses in the relevant frequency range, arises from their relatively small permeability (2 – 5). Good conductors with high permeability should, thus, produce enormous eddy current losses – and this is what they do, to be confirmed by the following measurements. To achieve this, we have inserted cylindrical cores made of different materials into the above mentioned coil: steel, nickel, copper (**Fig. 4.42** right)

♥ The denomination *copper-resistance* is disadvantageous here, because copper is also used for the coil core.
♣ The (nonlinear) remagnetization losses are also insignificant.

Copper is diamagnetic, its permeability differs only marginal from μ_0. **Steel** and **nickel** are ferromagnetic, their permeability is considerably higher than μ_0. Copper is a very good electrical conductor, Nickel has higher resistivity by a factor of 4, steel by a factor of 10 – 20. As can be seen clearly by the measured curves (Fig. 4.42, right) the high-frequency impedance increase with these metal cores is shallower than in air or in ferrites. The origin of this behavior is the **eddy currents**, which increasingly force the field lines out of the core with increasing frequency and, thus, decrease the inductance. **Fig.4.43** shows the internal field distribution for a steel cylinder (∅ = 5 mm, length = 14 mm) for three frequencies as well as the frequency-dependence of the magnetic impedance, which, as complex unit, consists of a real part (magnetic resistance) and an imaginary part (eddy current losses). The losses have to be imaginary because the magnetic resistance is generally defined to be real – different from *electrical* networks, where loss-resistances usually are defined to be real. However, these are only conventions, finally only orthogonality between effective and reactive power is necessary.

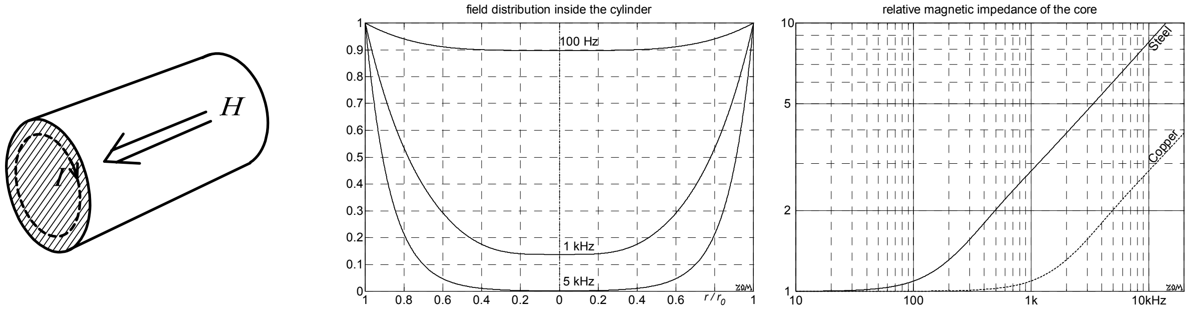


Fig. 4.43: An axial magnetic field *H* growing with time generates the eddy current *I* in the metal cylinder. This current will produce a circular magnetic field around itself, which is in opposite direction to the generating field and forces it out of the cylinder. The picture in the middle shows the radial distribution of the axial magnetic field in a steel cylinder (r_0=5mm), on the right the magnetic impedance normalized to low frequencies is depicted.

The basis of the calculations is **Maxwell'**s Laws in their differential form under the simplifying assumption that the electrical conductivity σ and the permeability μ are constant. For the conductivity this assumption is true, for the permeability actually not: the space and time-dependent flux-distribution leads to a space and time-dependent μ. The exact calculation in an anisotropic non-linear medium is, however, so complicated that simplification is necessary. Both Maxwell Laws now read as:

$$rot\,\vec{H} = \sigma \cdot \vec{E} \quad \text{and} \quad rot\,\vec{E} = -\mu \cdot \partial \vec{H} / \partial t$$ Differential form of Maxwell's Law

In cylinder coordinates *H* only exists in the axial direction, the field strength *E* exists only in the circular (azimuthal) direction and the rotation *rot* can, therefore, be simplified to:

$$\sigma \cdot E = -\partial H / \partial r \quad \text{and} \quad -\mu \cdot \partial H / \partial t = E / r + \partial E / \partial r$$ In cylindrical coordinates

Both formulas combined will yield **Bessel'**s Differential Equation, which can be solved for harmonic signals with complex units:

$$\frac{\partial^2 H}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial H}{\partial r} = \mu\sigma \cdot \frac{\partial H}{\partial t} \quad \rightarrow \quad \frac{\partial^2 \underline{H}}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \underline{H}}{\partial r} = j\omega\mu\sigma \cdot \underline{H}$$ Bessels Diff. Equation

The time differential operator $\partial/\partial t$ has been replaced by $j\omega$ (see system theory).

The **solution of Bessel's Differential Equation** for the radial distribution of the axially directed magnetic flux density $\underline{B}(r) = \mu\underline{H}(r)$ is:

$$\underline{B}(r) = \mu c \cdot J_0(kr) \qquad \text{with} \qquad k = (1-j)\cdot\sqrt{\pi f \mu\sigma} \qquad \text{or} \qquad k^2 = -j\omega\sigma\mu$$

Here c is an integration constant and J_0 is zero order Bessel's function of the first degree. The total magnetic flux that axially passes through the cylinder is given by the area integral over the cross section with r_0 = cylinder radius:

$$\underline{\Phi} = \int_0^{r_0} \underline{B}\cdot 2\pi r\cdot dr = 2\pi\mu c\cdot\int_0^{r_0} r\cdot J_0(kr)\cdot dr = \frac{2\pi\mu c}{k^2}\cdot\int_0^{kr_0} kr\cdot J_0(kr)\cdot dkr \qquad \text{Total flux}$$

The integration of Bessel's Function is carried out with $\int x\cdot J_0(x)\cdot dx = x\cdot J_1(x) + C$, where J_1 is a first order Bessel's Function of first degree. For the magnetic flux this yields:

$$\underline{\Phi} = \frac{2\pi c}{-j\omega\sigma}\left[kr\cdot J_1(kr)\right]_0^{kr_0} = j\frac{2\pi c k r_0}{\omega\sigma}\cdot J_1(kr_0) \qquad \text{Total flux}$$

The magnetic resistance is defined as quotient out of magnetomotive force and flux, the **length-specific** magnetic resistance R'_m is the quotient out of field strength and flux:

$$R_m = V_m/\Phi; \qquad R'_m = R_m/l = H/\Phi; \qquad \text{Magnetic resistance}$$

The length-specific magnetic resistance is calculated by dividing the field strength $H(r_0)$ by the flux Φ along the cylinder barrel; the result is complex and is, therefore, called the **length-specific magnetic impedance**:

$$\underline{Z}'_m = \underline{H}(r_0)/\underline{\Phi} = \frac{-j\omega c\sigma}{2\pi c k r_0}\cdot\frac{J_0(kr_0)}{J_1(kr_0)} = \frac{k}{2\pi r_0\mu}\cdot\frac{J_0(kr_0)}{J_1(kr_0)} \qquad \text{Length-specific impedance}$$

For very low frequencies k tends to zero and, using a series expansion of Bessel's Function, one will get as a (real) limit value $\underline{Z}'_m \to 1/r_0^2\pi\mu$, or the inverse of the cylinder cross-section and the magnetic conductivity. This means, that for low frequencies, there is no field displacement at all, the flux density is independent of position for the entire cross-section. However, with increasing frequency the magnetic flux is forced from the center to the boundary area (casing vicinity), the magnetic resistance (impedance) increases and the cylinder will become 'less magnetic' (see also chapter 5.9.2.4).

The field displacement calculated by Bessel's functions can qualitatively explain the impedance/frequency relation shown in Fig. 4.42. However, precise quantitative data are not possible because the (possibly tensor) magnet data are not known precisely and the metal cylinder is not percolated exactly axially. In contrast to the pickup calculations for metal cylinders, a finite elements (FEM) computation would be possible but, also for this case, the problem of insufficient material data remains.

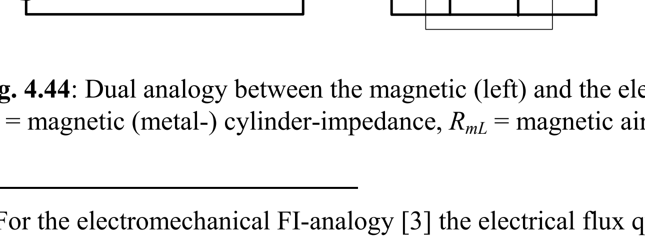
The (magnetic) field lines of the coil shown in Fig. 4.42 run partly in metal and partly in air. As already shown in chapter 4.6, one may visualize these magnetic networks by block diagrams, in **analogy** to electrical networks, in which resistors are displayed by rectangles. For the depiction of magnetic *loss* resistors there are no commonly defined symbols; in the following they are represented by rectangles enclosing a zigzag line (**Fig. 4.44**). The **magnetic impedance** Z_m (the inverse of which is the magnetic admittance Y_m) consists of real and imaginary parts: $\underline{Z}_m = R_m + jX_m$. It has to be kept in mind that the depicted loss resistors are imaginary – different from an electrical network. In order to project the networks onto one another using this analogy one has to define the flux quantity and the potential (difference) quantity [3]. The **flux quantity**♣ in electrical networks is the current, in magnetic networks it is the magnetic flux. The **potential quantity** is the electric voltage and the magnetomotive force, respectively. The flux quantity divides at nodes, where Kirchhoff's first law (or Maxwell I, respectively) is valid; in analogy, Kirchhoff's second law is valid for the potential quantity (or Maxwell II. respectively). Analogies that project flux quantities onto flux quantities create an **isomorphic** (equal structure) network; the projection of a flux quantity onto a potential quantity generates a **dual** network. Which is valid for **electromagnetic analogies**? Using the transformation mechanisms predominant for pickups (chapter 5) as orientation, one may find a projection of the magnetic flux to the voltage and of the current to the magnetic field strength – or **duality**. Written as equations:

$$U = N \cdot d\Phi/dt \qquad \text{and} \qquad N \cdot I = \oint H \cdot ds \qquad \text{Electromagnetic transfer formulas [3]}$$

The first formula represents the law of induction, the second the law of magnetomotive force (Ampere's law). Consequently, a magnetic series circuit will become a parallel circuit in the electrical block diagram. The differential occurring in the law of induction will be replaced by a multiplication with $j\omega$ for complex (sinusoidal) signals, which yields:

$$\underline{\boldsymbol{Z}}_{\mathbf{el}} = \frac{\underline{U}}{\underline{I}} = \frac{j\omega \cdot N \cdot \Phi}{\Theta / N} = j\omega \cdot N^2 \cdot \underline{\boldsymbol{Y}}_{\mathbf{m}} \qquad \Theta = \oint H \cdot ds = \text{Magnetic flux}$$

The magnetic and the electric impedance are thus reciprocal: The higher the permeability, the lower is the magnetic impedance and the higher the electric impedance. A real magnetic resistor will be projected into an imaginary electrical resistor (inductance, $\underline{Z} = j\omega L$), an imaginary magnetic (loss-) resistor will be projected into a real electrical resistor. The series connection of the magnetic real and imaginary part of the impedance $R_m + jX_m$ will become the parallel connection of the electrical resistance R and the inductance L; both are frequency-dependent. The magnetic in-series connection of the air resistance R_{mL} will become the parallel lying inductor L_L.



$$\underline{Y}_{el} = \underline{Z}_m / (j\omega N^2)$$
$$R = \omega N^2 / X_m$$
$$L = N^2 / R_m \; ; \quad L_L = N^2 / R_{mL}$$

Fig. 4.44: Dual analogy between the magnetic (left) and the electric network (middle).
$\underline{Z}_Z$ = magnetic (metal-) cylinder-impedance, R_{mL} = magnetic air resistor.

♣ For the electromechanical FI-analogy [3] the electrical flux quantity "current" will be projected to the mechanical flux quantity "force" in equal structure; the FU-analogy projects dually.

The magnetic cylinder impedance $\underline{Z}_Z$ for a **metal core** with length l and radius r_0 located in the center of the coil is:

$$\underline{Z}_Z = \frac{k \cdot l}{2\pi r_0 \cdot \mu} \cdot \frac{J_0(kr_0)}{J_1(kr_0)} \qquad \text{with} \qquad k = (1-j) \cdot \sqrt{\pi \mu \sigma \cdot f} \qquad \text{Magnetic cylinder impedance}$$

Here, μ is the (absolute) permeability of the core and σ is the electrical conductivity. Both the argument (kr_0) as well as the resulting Bessel function are complex. **Fig. 4.45** depicts the frequency dependence of the real and the imaginary parts of the magnetic cylinder impedance. If the metal cylinder were the only magnetic resistor in the (closed) magnetic loop, one would obtain 5.9 H for the low frequency case, as shown in the Fig. 4.46 (left). However, as for the cylinder coil under consideration, the field lines close over a long air distance and an air resistor also has to be taken into account.

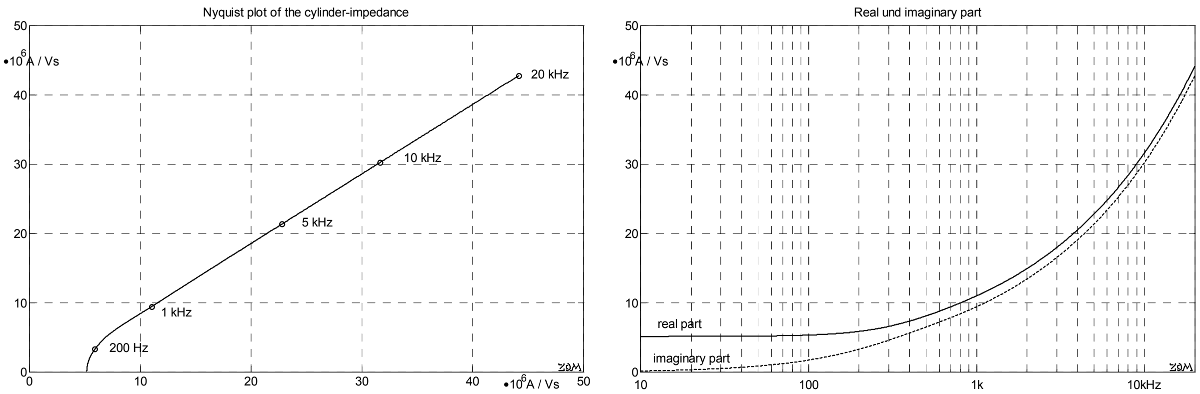


Fig. 4.45: Frequency-dependence of the complex magnetic cylinder impedance $\underline{Z}_Z$, $\mu_r = 110$, $\sigma = 5e6$ S/m.

If one considers, in a simple magnetic equivalent circuit, a series connection of core and air resistance (Fig. 4.44), this will reduce the absolute value of the inductance as well as its frequency dependence (**Fig. 4.46**). This simple model is well suited as long as the ferromagnetic metal core can sufficiently focus the field running through the coil. For small μ_r, however, a considerable part of the inner magnetic field flows within a kind of **hollow cylinder**, i.e. between core and average coil diameter. The magnetic resistance of this hollow cylinder is located parallel to $\underline{Z}_Z$ in the magnetic block circuit, hence, in the electrical equivalent circuit in series with the parallel connection of R and L (**Fig. 4.47**).

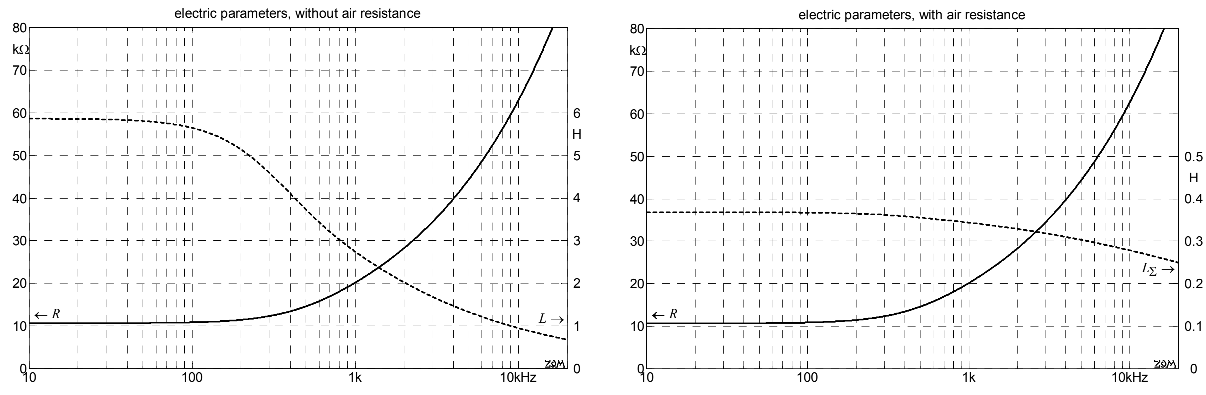


Fig. 4.46: Frequency dependence of R and L (left) as well as R und L // L_L (right) from Fig. 4.44 ($N = 5500$).

It is possible to explain every impedance/frequency curve in Fig. 4.42 with a good precision using this extended equivalent circuit diagram (**Fig. 4.47**). The magnetic resistance of this "hollow cylinder" is real and it is mapped onto the inductor L_{HZ} – by definition its *electrical* impedance is purely imaginary. The values of R and L are, as explained by Fig. 4.44, frequency-dependent.

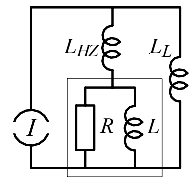


Fig. 4.47: ECD

For the **magnetic pickup** the magnetic eddy current losses have the following consequences: 1) The pickup resonance is damped not only by the wire resistance of the coil but also the ferromagnetic core inside the coil. 2) The inductance of the coil is frequency-dependent and decreases towards higher frequencies. A higher order *RL*-circuit can be employed in the block diagram as an alternative to the frequency-dependent inductance (see chapter 5.9.2.3). The different geometries and the diversity of the material parameters produce different damping and inductance frequency-dependencies. Using this, the pickup designer can purposely influence the frequency transfer characteristics.

Fig. 4.48 shows the impedance/frequency curves taken with a measuring coil (N = 5500, Fig. 4.44). The highest inductance is created by the ferrite rod, whose isolated elemental magnets do not allow eddy currents in this frequency range. The permeability of the humbucker screw, made of undefined steel, is practically the same for low frequencies (300 Hz), however, due to high eddy currents, its inductance decreases. The humbucker cylinder ("slug") has a somewhat smaller inductance for lower frequencies but also smaller eddy-current losses. Alnico-magnets are practically free of eddy currents; the magnetically weaker Alnico 2 has a higher permeability as compared to Alnico 5 (Alnico 500), resulting in a lower pickup resonance (for otherwise equal parameters).

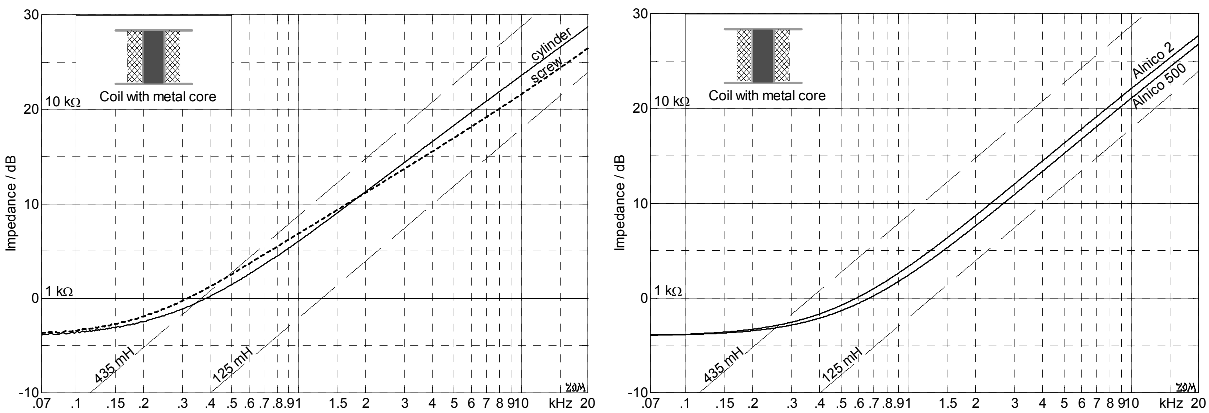


Fig. 4.48: Impedance/frequency curves, taken with the measuring coil; core dimensions 5x14 mm. "Cylinder" means the metal cylinders (= slugs) commonly used for humbuckers, "screw" depicts the humbucker screw (5.9.2.6).

Elaborate details for the construction of single-coil and humbucker pickups, as well as their technical data are summarized in chapter 5.

4.11 Magnetic Field Forces

Magnetic forces are the most obvious effect of the magnetic field: If one places a *ferro*magnetic material into the field of a permanent magnet, it will be drawn towards one of the magnetic poles. Magnetic forces also act for *para*- and *dia*magnetic materials, however, they are barely detectable. Only when the string is composed of a ferromagnetic material its vibration can be effectively detected by a magnetic pickup, because only then will the string significantly change the magnetic flux, so that a sufficiently high voltage is induced. At the same time, however, the magnetic forces will change the vibration mode of the string – the generation of a voltage in the pickup is, thus, not free of retroactive effects.

Theoretical physics does not view the electrical and magnetic fields as independent and self-contained conditions in space, but rather combines both phenomena into a unique field theory. Forces between stationary charges have to be treated differently from charges in motion: a relativistic approach is necessary even for small velocities. However, for the pragmatist says that *veritable is only what is appropriate for the act* and he gains the winning tender, in this case. The unique field theory is elegant but, for the present considerations, classical electrical engineering theory is sufficient and describes – as shown in the following – the force effects as independent phenomena.

4.11.1 Maxwell's Force

A ferromagnetic string brought into a magnetic field experiences a magnetic force. Here, it does not matter whether the string approaches the north or the south Pole; in both cases it will be attracted. The larger the field strength the larger the attractive force. The force, however, does not generally act in the direction of the of the field strength – and likewise also not generally in the opposite direction. Most simply one can interpret the magnetic force as a surface force that affects the entire surface of the string. Hereby it is understandable why an iron ball will stay at rest if brought into a *homogenous* magnetic field: the drag forces acting on both halves of the ball are balanced and the resulting sum of forces is zero. The fact that an iron ball in the field of a horseshoe magnet is nevertheless attracted by one of the poles is due to the inhomogeneity of the field. Only in a very theoretical middle position could an instable balanced condition be constructed; in every other position one of the two forces dominates and will accelerate the ball. This is completely different for the Coulomb force (4.11.6): a charged Styrofoam ball will also be accelerated in a *homogenous* electrical field.

The magnetic force effect may be very obvious; however, it still remains difficult to understand its underlying mode of action. Around the beginning of the 19th century magnet scientists still had the opinion that magnetized bodies would act on each other by a **long-distance effect**. This fact, that even an intermediate vacuum could not prevent this long-distance effect, lead to the conviction that the intermediate space was not involved and that the magnetic forces would directly act on the bodies without changing the space in between. The first person to define the **concept of the field** was Michael **Faraday** at around 1830, which changes the space between the bodies by force lines (**near-field theory**): the space itself will now become the medium and transmitter of the force.

James Clerk **Maxwell** (1831 – 1879) extended Faraday's ideas into a comprehensive electromagnetic field theory. A field is assigned to every point in space, which is defined by its **field quantity**. For the magnetic field these quantities are the field quantities *H* and *B*. The permanent magnet of the pickup generates an electromagnetic field that acts on other bodies (e.g. on a string) and produces forces there. However, the now magnetized string will also produce a field that acts on the permanent magnet. The generation and changes of these fields happen in the pickup practically without delay.

A mechanical stress state can be assigned to every point in the magnetic space within a very general force effect theory. The theory of elasticity distinguishes between **normal stress** (force perpendicular to the area) and **shear stress** (force runs within the area). For example, if a steel cylinder is stressed in the axial direction a tensile stress is generated which will elongate the cylinder. At the same time it will become a little bit thinner, because a compressive stress acts in the radial direction (lateral contraction). On the other hand, shear stresses are generated by shearing-off a whisker, which are also called **shear strains**.

A general quantity for the characterization of the mechanical state of stress is the **stress tensor**: it describes the mechanical stress load that a differential small volume of the string is exposed to. By integration over all these string volumes (mathematically formulated by Gauss's integral law) one will arrive at surface forces that act in a radial direction with respect to the string's surface. Since two magnetically very different materials converge at the string-air interface, one can obtain a very simple approximation for the normal force per area (*F/S*):

$$\frac{F}{S} = \frac{B^2}{2\mu_0} \qquad \mu_0 = 1{,}26 \cdot 10^{-6}\,\frac{\mathrm{Vs}}{\mathrm{Am}} \qquad 1\ \mathrm{VAs} = 1\ \mathrm{Nm}.$$

The area-specific force is proportional to the square of the flux density. For *B* one has to take the value that will result at the string surface and not the value which will be measured without the string. The value at a distance of approx. 2 mm in front of a pickup magnet without the string will be 20 – 50 mT, while it will be approx. 200 mT including the string. The string, as a consequence of its good magnetic conductivity, "sucks" the surrounding field lines as it were and, thus, increases the local flux density. As a rough approximation one will get 48 mN for the magnetic force for a string area of 3 mm² and 200 mT flux density. A precise calculation is difficult, because in this case the three-dimensional field distribution in two non-linear media would have to be determined. In contrast, **measurements** convey a sufficiently precise picture: for this a magnetic pickup was moved towards a steel wire (0.7 mm diameter) and the resulting magnetic force was measured (**Fig. 4.49**). Forces of 10 to 40 mN are detected for common separations – a good confirmation of the theoretical estimation. For a typical humbucker (e.g. Gibson '57-Classic) the forces are smaller.

In comparison to the string tension force (50 – 200 N) the magnetic forces are very small; the lateral string displacement caused by them is less than 0.1 mm. Nevertheless, the effect of the magnetic field must not be totally ignored, because its stiffness changes the frequency of the string. The nearer the string comes to the magnet, the more it is pulled. The differentiation of this force/distance relation will yield a distance-dependent stiffness of –1 ... –30 N/m; in contrary to common springs it is negative. The numbers are to be interpreted as guiding value; the measurement precision is only moderate.

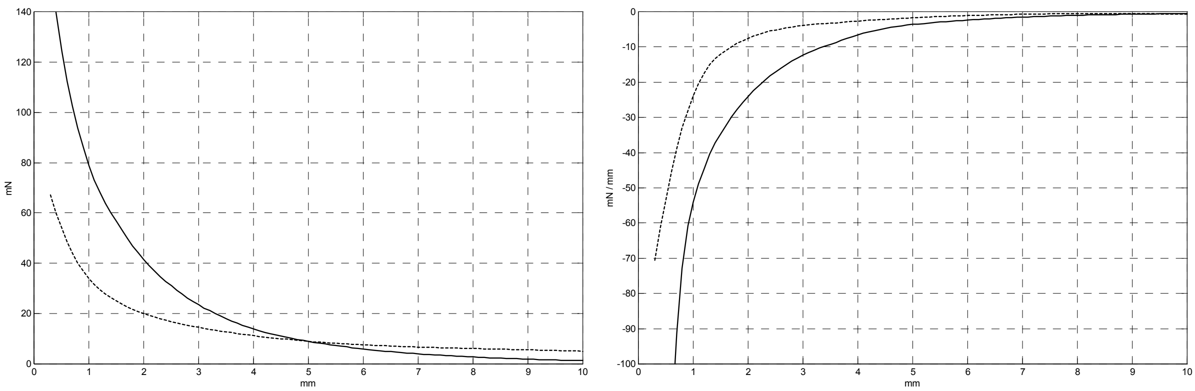


Fig. 4.49: Magnetic force as function of the clearance. Alnico-5, singlecoil (——), Gibson-humbucker (---). In the right picture the differential stiffness is shown as a function of the clearance.

However, the **negative magnetic field stiffness** not only affects the vibrations normal to the fretboard; for vibrations parallel to the fretboard the string/magnet distance is practically constant, the magnetic field stiffness is therefore negligible♣. For vibrations normal to the fretboard the negative field stiffness generates a decrease of the mechanical stiffness and, hence, a decrease of the partial tone frequency of the string. The effect is not dramatic, but audible for strong magnets: if the magnet is moved closer the tone frequency drops. However, every vibration of the string will occur as spatial wave, not as plain transversal wave. Even if a *plain* vibration is prevalent shortly after plucking, mode coupling in the supporting points and, last but not least, the magnetic field will cause a rotation of the original vibration plane. The rotation frequency is low (some Hertz) and beat-frequency-like **amplitude variations** will evolve in the pickup signal (4.11.3).

In this way the magnet does not just change the tone frequency but also the tone color of the vibrating string. Whether this is good or bad depends on subjective assessment criteria. Many guitarists have the opinion that the chorus-like beat frequencies of a Stratocaster belong to the typical sound of this guitar – as long as they are not too dominant. The assumption (or "certainty" of expert authors) expressed in several books that "the harmonics are slightly detuned in comparison with the fundamental*s*" is incorrect: the **fundamental** will be detuned the most. One can ask, since 2001, why it is suddenly referred to in the plural, and who feels that the announcement: "*further handbooks are in preparation*" is mere threat. However, this is just how they are, those string fundamentals.

4.11.2 Field-Induced Deviations of the Tone Frequency

When a magnetic pickup approaches a string, three effects can be anticipated: The **tone frequency** decreases, chorus-like **beat frequencies** evolve and the **amplitude** changes. The frequencies, especially the fundamental, will decrease due to the negative field stiffness, which can be audible for large values. The detuning between the fretboard-normal and fretboard-parallel vibrations induces beat frequencies; the altered frequency relationship between the partial tones in the subsequent non-linear systems causes additional partial tones, which can further increase the chorus impression. True damping, i.e. removal of vibrating energy, occurs only to a negligible extent. Firstly to the tone frequency:

♣ If the string is located substantially beyond the magnet axis, both vibrating planes are affected.

The resonance frequency of a vibratory mass-spring-system depends on the square root of the spring stiffness. The stiffness caused by the magnetic field is negative because approaching the magnet does not need a force in the direction of the movement (as it is for every common spring) but in the opposite direction: the magnet does not need to be pushed towards the string with force but, to the contrary, must be held back. The negative stiffness which is acting thereby decreases the total stiffness of the string and reduces the frequency. The frequency-dependence of this effect or which partial tones may be affected can be investigated with the **conduction-analogy**. Here, the mechanical system is described by an analogous electrical circuit with the analogies: force/current, velocity/voltage, spring/coil, mass/capacitor [3]. The principle effect can be investigated for an undamped plain transversal wave which is ideally reflected at a solid mounting. The string corresponds to an electrical conductor shorted at the end and whose length is large in comparison with the wavelength [e.g. Meinke]. The corresponding *mechanical* input impedance $\underline{Z}_E$ depends on the wave resistance Z_W, the terminating impedance ($\underline{Z}_{termination} \to \infty$, because of velocity $v = 0$), the conductor length l, on the frequency f and the phase velocity c. All of these system parameters can be attributed to the mechanical quantities by the analogy-laws: the string tension force Ψ, the string density ρ, the string length l and the string cross section area A.

$$\underline{Z}_E = \frac{Z_W}{j \cdot \tan \beta l}; \qquad \beta = \frac{\omega}{c} = 2\pi f \sqrt{\rho A / \Psi}; \qquad Z_W = \sqrt{\rho A \Psi} \qquad \text{Conduction}$$

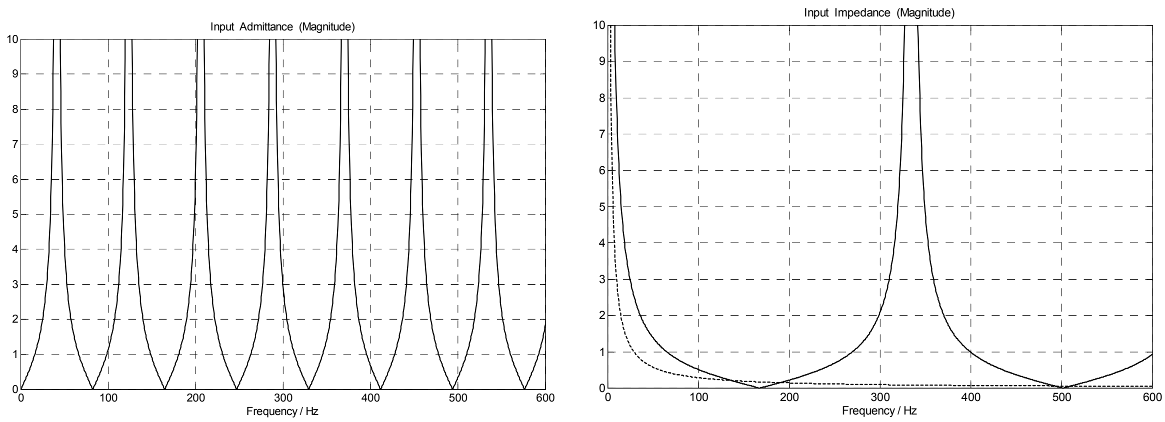
If one assumes a solid mounting for *both* ends of the string ($\underline{Z}_{el} = 0 \;\hat{=}\; \underline{Z}_{mech} = \infty$), the Eigen-frequencies (partial tone frequencies) of the string are the poles of the tangents-function, i.e. at integer multiples of the basic frequency $f_G = c/2l$. The reciprocal of the base frequency is the transit time over $2l$, i.e. from the beginning of the conductor to the reflecting end and back. In order to introduce the influence of the negative field stiffness into the conduction model, one divides the string into two consecutive conductors: a first conductor of length l_1 from the saddle to the pickup and a second conductor of length l_2 from the pickup to the bridge. The mechanical termination impedance $\underline{Z}$ of the first conductor is the sum of the input impedance $\underline{Z}_2$ of the second conductor and the stiffness impedance $\underline{Z}_S$: The input impedance $\underline{Z}_1$ of the first conductor (viewed from the saddle) is thereby given by:

$$\underline{Z}_1 = \frac{\underline{Z} + jZ_W \cdot \tan \beta l_2}{1 + j\,\underline{Z}/Z_W \cdot \tan \beta l_2} \qquad \underline{Z} = \underline{Z}_2 + \underline{Z}_S \qquad \underline{Z}_S = \frac{s}{j\omega}$$

The Eigen-frequencies are located at the poles of the impedance function, i.e. at $\underline{Z}_1 \to \infty$.

Of course the input-impedance can also be calculated from the location of the bridge with the same result. For a first check the magnetic field stiffness can be taken to be zero ($\underline{Z}_S = 0$, $Z = \underline{Z}_2$) which indeed gives the partial tone frequencies at multiples of 82,4 Hz using the data of the E_2-string. As can be expected, a spring with stiffness zero cannot produce any changes. For every stiffness s different from zero the absolute value of $\underline{Z}_S$ will tend to zero with increasing frequency ($\underline{Z}_S = s/j\omega$), from which immediately follows, that the magnetic field stiffness can only detune low-frequency partials. As the field stiffness is negative the partial frequencies will *decrease*.

Figure 4.50 upper left shows the calculated (mechanical) input admittance of an E_2-string (82,4 Hz). The admittance is the inverse of the impedance; its zero points lie at the poles of $\underline{Z}_E$. The upper right picture shows the absolute value of the impedance of a 16 cm long part of the string located between bridge and magnet, as well as the absolute value of a magnetic field stiffness (–180 N/m). Its value was chosen to be unusually high to depict the effect more clearly. In the lower left plot the effects of the field spring on the admittance of the entire string are shown: especially the first and the second partial are detuned. For the calculation a characteristic wave impedance of 0.7 Ns/m was chosen; the length of the string is 65 cm and the magnet is located at a distance of 16 cm from the bridge. The frequencies of the partial tones are at the zero positions of the admittance. No dispersion was modeled.



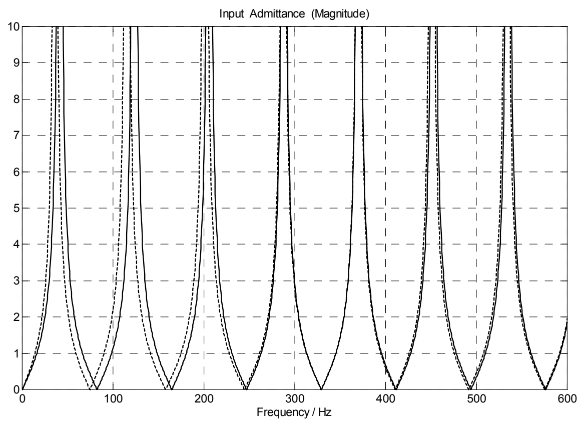


Fig. 4.50: Frequency dependences of the magnitudes.
Upper left: Admittance magnitude of an E_2-string.
Upper right: Impedance magnitude of the field stiffness (---), absolute value of the impedance of a 16 cm long part (—); both impedances are to be added.
Lower left: Admittance magnitude as in the upper left plot (—); Admittance magnitude including additional field stiffness (---).
Zero positions → frequencies of partial tones.

In addition to the calculations measurements of an E_2-string are shown in **Fig. 4.51**. A Fender E_2-string (3150, 1.1 mm diameter) was mounted in an Ovation solid-body guitar (EA-68, piezo-pickup) and the piezo-signal was analyzed. The magnetic forces were generated by an 18 mm long Alnico-5-Magnet (5 mm diameter) positioned relative to the string at a distance of 16 cm from the bridge.

Fig. 4.51 shows that a precise frequency analysis is problematic: the resulting detuning is only several Hertz, so that a frequency resolution smaller than 1 Hertz would be desirable. The string vibration, however, cannot be considered as stationary within the necessary time window (more than 1 s). The chosen DFT-windows represent a compromise between time and frequency resolution (analysis was done with the CORTEX-software *Viper*).

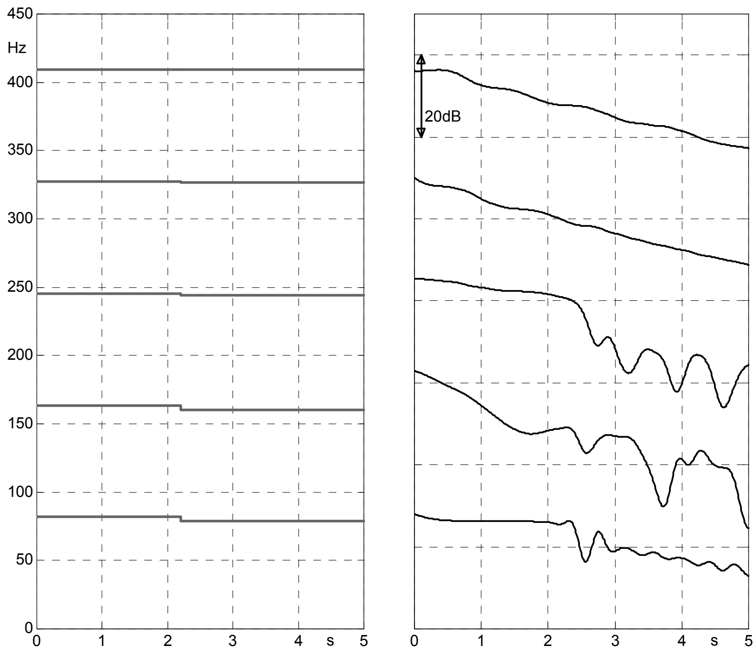


Fig. 4.51: Spectrogram (left) and partial tone level progress (right) of the vibration decay of an E_2 string. At 2.2s a magnet was approached to the vibrating string.
The frequencies of the first and second harmonic decrease from 2.2 s onwards . For the third harmonic one can mainly detect a level change, the fourth and the fifth harmonic remain unchanged (holds also for higher harmonics).

4.11.3 Field-Induced Amplitude Variations

The measurements and conduction model show, concordantly, that the permanent magnet will detune the lowest harmonics. The detuning will happen mainly for the fretboard-normal vibrations; field changes parallel to the fretboard and thus parallel to the magnetic pole surface only develop weakly. For the spatial vibration this means that there are two spatially orthogonal string vibrations with different frequencies which, after superposition, produce **beat frequency-like level changes**. If one denotes the fretboard-normal component with y and the fretboard-parallel component with x, one gets for the total amplitude ξ in vector-notation:

$$\vec{\xi} = \begin{pmatrix} \hat{x} \cdot \cos(\omega_1 t) \\ \hat{y} \cdot \cos(\omega_2 t + \varphi) \end{pmatrix}$$

$\hat{x}$ = Amplitude of the x-component
$\hat{y}$ = Amplitude of the y-component

For single frequency vibrations ($\omega_1 = \omega_2$) a point on the string moves according to the amplitude-relation $\hat{y}/\hat{x}$ and the phase shift φ along a line in an ellipse or a circle* (**Lissajous** figures). However, if both frequencies are not equal the figures above alternate with weak transitions. The time-dependent change of the curve is apparent when one transforms for small frequency differences:

$$\omega_2 t + \varphi = \omega_1 t + \Delta\omega t + \varphi = \omega_1 t + \varphi(t)$$

The x as well as the y-vibrations contain $\omega_1 t$, however, for the y-vibration an additional time-dependent (slow) phase-shift $\varphi(t)$ exists. A sensor that only detects the vibration exactly normal to the guitar body will, however, not be affected by the curve changes because $\hat{y}$ is time-invariant.

* Line and circle are special types of the ellipse.

Real sensors cannot be expected to exhibit such a perfect direction sensibility: common magnetic pickups are indeed the most sensitive for fretboard-normal vibrations; however, for fretboard-parallel vibrations the sensitivity will not be zero but approx. 1/10. The voltage which is generated is, thus, a combination of x and y vibrations which can, for the simplest case, be depicted as a **linear combination**:

$$u(t) = U\left(\cos(\omega_2 t + \varphi) + k \cdot \cos(\omega_1 t)\right) \qquad k = \text{relative } x\text{-ratio}$$

The commonly known formula for the beat frequency is obtained for k = 1 whereas for k << 1 the signal can be approximately regarded as a mixture of frequency and amplitude modulation. A cosine-like frequency modulation for a small modulation index can be represented, to a good approximation, by three spectral-lines [3]:

$$u_{FM} = U\left[\cos(\omega_2 t) - \frac{m}{2}\cos\left((\omega_2 + \Delta\omega)t\right) + \frac{m}{2}\cos\left((\omega_2 - \Delta\omega)t\right)\right]$$

If this FM signal should become amplitude modulated, the AM has to be applied to each of the three spectral components. By neglecting the $m^2/4$-terms (because $m \ll 1$), the lines at $\omega_2 + \Delta\omega$ compensate, while the lines at $\omega_2 - \Delta\omega$ add:

$$u = U\left[\cos(\omega_2 t) + m \cdot \cos\left((\omega_2 - \Delta\omega)t\right)\right] \qquad \omega_2 - \Delta\omega = \omega_1$$

This signal equates to the above mentioned linear combination for $\varphi = 0$; corresponding transformations are possible for other phase shifts. Hence it has been shown that for x and y vibrations with $k = 1$ a beat frequency, and for $k \ll 1$ a mixture of AM and FM, will result. This result can also be derived from the projection of the sum of two pointers with different frequency. If one assumes, for example, that the pickup for the y oscillations is eight times more sensitive than for the x oscillations ($k = 0.125$) then, for $\hat{y} = \hat{x}$, the amplitude of the pickup voltage changes by ±12.5%, or ± 1 dB. The modulation frequency corresponds to the difference frequency, which is the detuning caused by the magnet (e.g. 1 Hz). The amplitude relation $\hat{y} = \hat{x}$ means that the string vibrates at an angle of 45° with respect to the fretboard. The amplitude modulation effect will decrease for larger angles (normal to the fretboard) and for smaller angles (→ fretboard-parallel) it increases, until at arctan(1/8) = 7° a precise beat frequency is reached: The level change here is theoretically unlimited.

The linear combination is only a simple model for the description of time-variant level fluctuations. For the magnetic pickup the induced voltage depends on the **non-linear** relationships of the x and y velocities, which will result in additional sum and difference tones. However, as this will not result in completely different effects, we have dispensed with a precise investigation. An additional effect, which has also not been taken into consideration acts at both **string mountings** (bridge / saddle). Both mountings are idealized as rigid, but show a direction dependent compliance. As a consequence, the reflection factor has to be defined including all modes: a pure y vibration will also be reflected, to small extent, in the x direction and vice versa. For example, if the string is plucked exactly normal to the fretboard, after a certain time there will be fretboard parallel component which will yield amplitude variations in the pickup; the magnetic field can enhance or diminish them.

In addition, the (predominantly) fretboard normal **magnetic field** can induce a **rotation** of the vibrating plane when it is not exactly fretboard-normal or fretboard-parallel: for an inclined vibrating angle the string will experience a stronger pull at the turning point closer to the magnet than at the turning point further away. By reducing the magnetic force into coplanar and orthogonal parts one will get an angular force that tries to align the string (along fretboard-normal direction).

Finally, one has also to consider that the field stiffness is **non-linear**: the absolute value of the stiffness increases with decreasing distance. A simulation with a non-linear conduction model results in weak beat frequencies, even for exact fretboard-normal vibrations, whose fractional variation amplitude is dependent on the input signal amplitude.

In summary: Already without magnetic fields, sound level deviations are generated that develop differently for each partial tone. They originate from anisotropic mounting reflections, i.e. mounting impedances that depend on the oscillation direction and mode-coupling. The magnetic field detunes the fretboard-normal vibration component which might enhance or reduce the existing deviations. Non-linearities occurring in the mechanics and during electro-mechanic transformation will create additional sub-lines in the spectrum so that, in summary, a complicated level characteristic might develop for each partial tone.

Fig. 4.52 shows the selectively measured characteristic of the partial tones of the E_2-string. The recordings were performed with the built-in piezo pickup without a magnetic field. The differences between both pictures originate from the plucking technique as well as from the non-identical guitar positions and, possibly, slightly different guitar tuning and temperature. In the course of these first orienting investigations it became clear that the guitar should not be placed somehow on the thigh but must be supported in a defined way. Appropriate frame conditions are “in vivo” (guitar hanging from the guitar strap, fret-hand defined at the neck), and “in vitro” (guitar attached at the strap-pin, no damping at the neck).

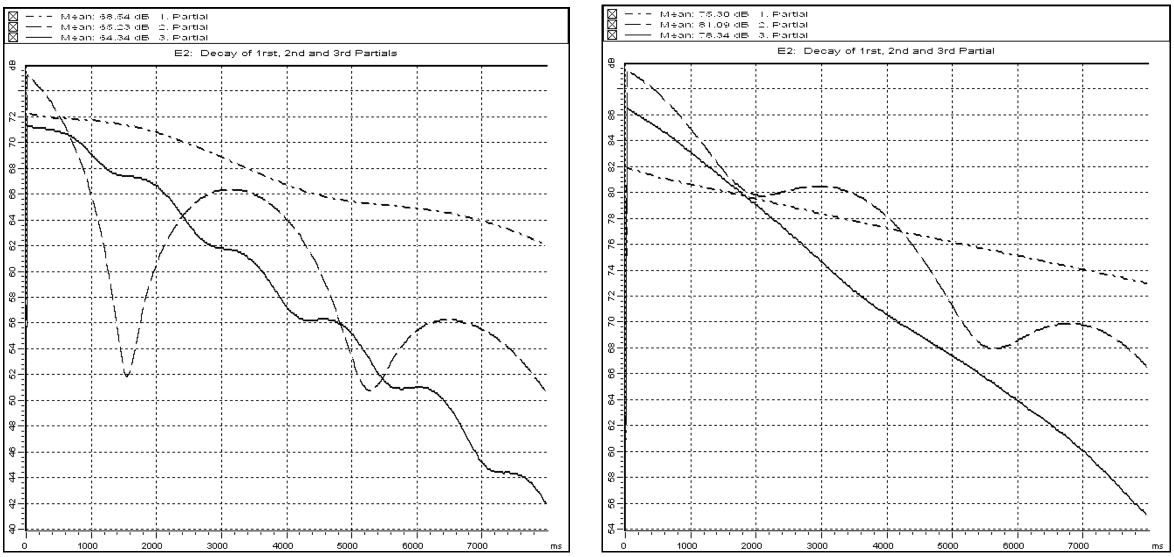


Fig. 4.52: Decay of the first three partial tone levels after plucking (left) or fretboard-normal excitation pulse (right) for an E_2-string with no magnet and an Ovation EA-68 piezo-pickup. The recordings were taken on different days.

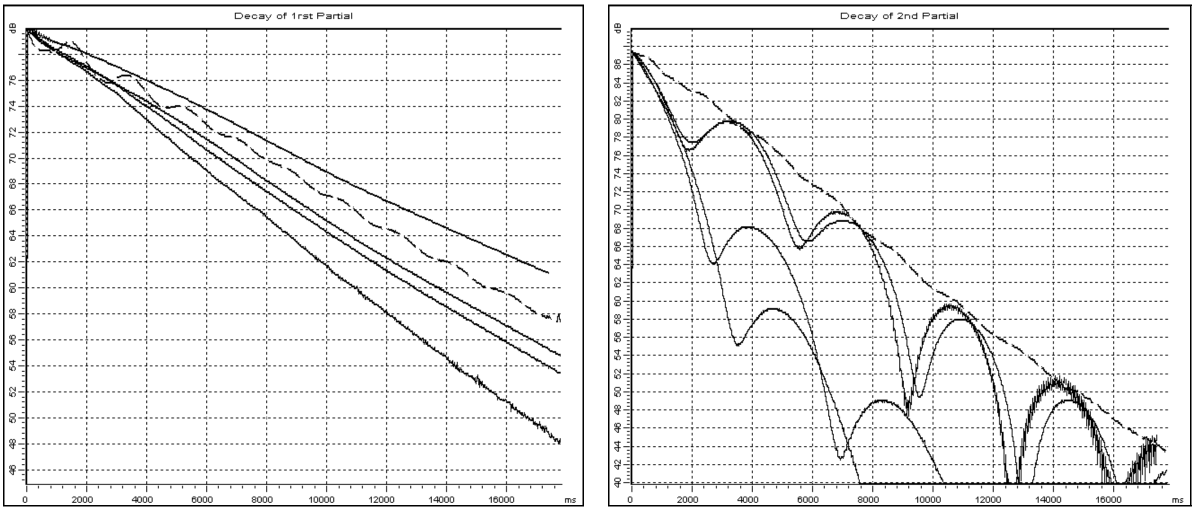


Fig. 4.53: Decay of the first two partial tone levels after fretboard-normal excitation pulse for an E_2-string, with an Ovation EA-68 piezo-pickup. Continuous lines: without magnet, dashed lines: Alnico-5-magnet in neck pickup position for a 2.5 mm distance between the string and magnet. Left: first partial, right: second partial.

Figure 4.53 shows the decay of the first two partial tones. The continuous lines were taken without a magnetic field. The upper curve, with the slowest decrease, shows the level decay of the undamped neck whereas the lower three continuous curves belong to measurements that were made with the fret-hand holding the neck in different ways without touching the strings. The dashed line was taken without neck-damping but with a magnetic field (Alnico-5-magnet placed 16 cm from the bridge). A strong influence of the fret-hand on the decay-characteristic (sustain) is observed in both measurements. The hand primarily acts as a damping resistance removing vibration energy. The level decays linearly with time for the first partial tone (left picture) without a magnetic field (exponential tension envelope curve), whereas a slight level oscillation occurs with a magnetic field. The second partial tone is completely different: There are intense level oscillations without a magnetic field, whereas there is a nearly oscillation-free decay with a magnetic field. **Fig. 4.54** shows similar results for fretboard-parallel excitations (both with magnetic field).

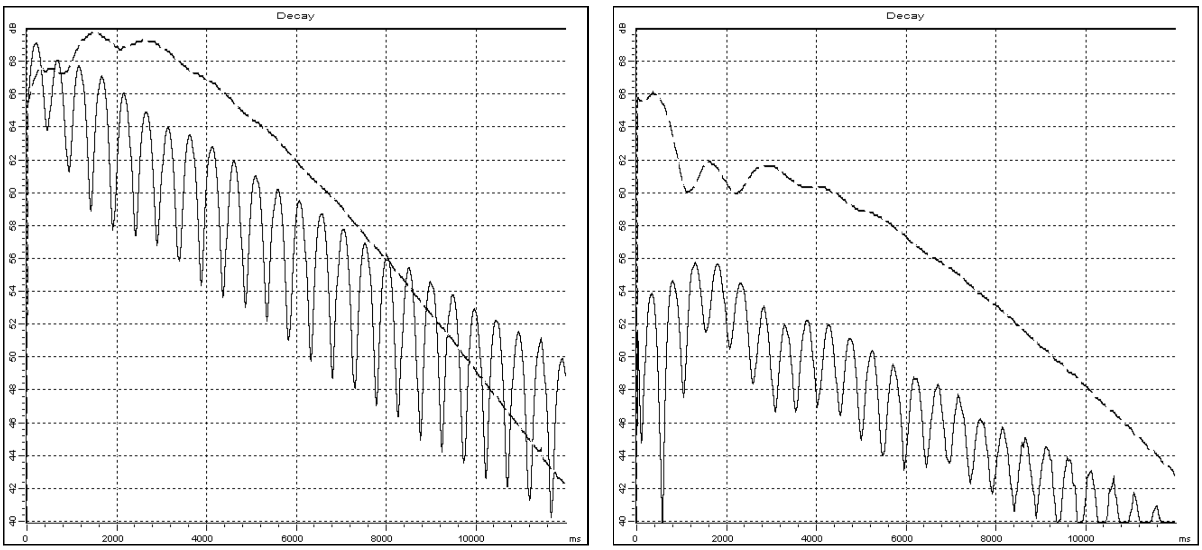


Fig. 4.54: Decay of the first (continuous) and third (dashed) partial tone after fretboard-parallel excitation using an Alnico-5-magnet in the neck pickup position. The only difference between both pictures is a slightly different plucking direction.

4.11.4 Field-Induced Damping

Pickup magnets are rumored to disturb the decay and to deteriorate the sustain of the string. Indeed, as shown in chapter 4.11.3, the magnetic stiffness can induce changes in the vibrational parameters; at small magnet distances these changes are also audible. However, an (ideal) spring is not able to extract energy from an oscillating system. If one pushes an ideal spring (with positive stiffness) it will store energy. However, after expansion this energy is returned entirely and without loss. In information technology one speaks of **reactive energy** in contrast to **active energy**, which is “lost” in a frictional resistance. The term “energy loss” can, of course, not just be viewed globally: in reality energy cannot be lost; however, it will be transformed irreversibly into thermal energy due to the frictional resistance and is no longer available for the oscillating system.

However, energetic considerations at a pickup are dangerous and may lead to the wrong conclusions: a pickup does not transform the vibration energy of a spring into electrical energy; rather it partakes from one component of the oscillation. Customary pickups mainly react to fretboard-normal vibrations. If a magnet would rotate the vibration plane of the string from fretboard-normal to fretboard-parallel this would not affect the vibration energy – nevertheless the pickup output voltage would decrease. Fortunately, this rotation occurs rather in the opposite direction (from fretboard-parallel to fretboard-normal); in this case the magnet will indeed increase the pickup output voltage, however, without increasing the vibrational energy.

At one place, however, real power is necessary: the voltage delivered by the pickup heats the ohmic resistors of the electrical circuit, and this real power has to be drawn from the vibration of the string, because the magnetic pickup is a passive transducer [3]. In addition, the so-called active pickups are passive with respect to their transformation process; in this case only the first amplification stage is located at a different place. The **ohmic resistors** in the electrical pickup load circuit are the volume potentiometer, the tone potentiometer, the amplifier input resistance and the coil resistance. The cut-off frequency of the 250 kΩ and 50 nF series connection (tone-pot) is 13 Hz, for higher frequencies the capacitor is approximately a short circuit. Both potentiometer resistances and the amplifier input resistance are in parallel for the standard circuit and, therefore, the result for the total resistance is 100 – 200 kΩ. Further, one has to add the coil resistance (4-15 kΩ). For the pickup/cable resonance one would have to consider a load transformation for the exact calculation, the following orienting calculation assumes 100 kΩ for simplicity. According to this calculation a pickup, that generates 100mV produces a real power of $P = U^2/R = 0.1\ \mu\text{W}$. This is very small but must be viewed relative to the string energy.

The kinetic energy of a mass differential dm is $dmv^2/2$. Here, v is the velocity of the differential mass. The **kinetic energy of the string** will be highest at the transit through the rest position. Integration over the total length of the string (with sinusoidal length-dependent velocity) yields $W = mv^2/4$, with m = mass of the entire string and v = velocity at rest position.

A typical Stratocaster pickup will generate an effective voltage of U = v·0.186 V for a 0.66 mm solid string at a magnet-distance of 2 mm; the velocity v has to be inserted as an effective value in m/s. However, the velocity is not the one stemming from the energy-formula, rather is it the velocity of the string *above* the pickup. For an oscillation of the first partial the maximum of the velocity is located in the middle of the string (12th fret); above the neck-pickup v is only 0.69 times as big. In addition, one has to bear in mind that in the energy formula the amplitude of the velocity is depicted, whereas for the computation of the voltage the effective value of the velocity is necessary. This will yield for the mechanical energy W and for the electrical power P:

$$W_{mech} = \frac{1}{4} m \hat{v}^2 \qquad P_{el} = \frac{U^2}{R} = \frac{(0.186 \cdot 0.69 \cdot \tilde{v})^2}{100\,\mathrm{k}\Omega} \qquad \frac{P_{el}}{W_{mech}} = \frac{3.3 \cdot 10^{-7}}{m} \cdot \frac{\mathrm{kg}}{\mathrm{s}}$$

The power P is the quotient out of the energy loss dW and the duration dt (power is energy over time), the relative energy loss is thus $dW/W = Pdt/W$. Using 1.78 g for the mass of the string, the relative energy loss per second is 0.019 %. The time dependent damping value, the **decay-rate** D thus will be:

$$D = 10 \lg \frac{W}{W - \Delta W} \frac{\mathrm{dB}}{\mathrm{s}} = -10 \lg\left(1 - \Delta W / W\right) \frac{\mathrm{dB}}{\mathrm{s}} \approx \frac{10}{\ln 10} \cdot \frac{\Delta W}{W} \frac{\mathrm{dB}}{\mathrm{s}} = 4.34 \frac{\Delta W}{W} \frac{\mathrm{dB}}{\mathrm{s}} \quad ♣$$

Here, ΔW is the energy-loss over 1 s, which will be computed as $P \cdot 1$ s. With the above string one will get a decay rate of 0.0008 dB/s. This is the level decrease resulting from the electrical damping. Even if one assumes much more efficient pickups with e.g. ten times larger transformation coefficient, this effect is still minimal and can surely be neglected compared to other damping mechanisms.

This seems to result in very simple conditions: the magnetic field acts as a spring mainly on the lower partials and the electrical losses are negligible. However, it is not quite that simple. The problems are already present in the **measurement of the decay curves**. It is relatively simple to choose the appropriate DFT-windows that enable a sufficiently fast and selective measurement of single partials. For most of the measurements with the CORTEX-software *Viper* the 50-dB-Kaiser-Bessel-window with N = 4096 and zero padding = 2 turned out to be well suited. The decay lines of the partial tones, however, are often curved and, thus, hamper the modeling. In **Fig 4.55** the level trends for the E_2 string are depicted, taken without and with magnetic field. How can the decay (the sustain) be defined with <u>one</u> number? As a level change within the first second? Every time interval chosen appears to be arbitrary. The guitarist will not be fussed about the functional decay of the vibrational level, however, for basic research it will rather play an important role whether the level decay will be caused by dissipation or by exchange of vibrational energy. For the case of rapid beat frequencies (right picture) it seems to be relatively simple to derive a time-dependent envelope function from of the maxima. But if the beat frequency period lasts for ten seconds or longer, the measurement can become impossible: Until the next beat frequency maximum the oscillation may possibly have become too small due to other damping mechanisms. It is also not particularly practical to extract average values from a 30 second level decay because in music tones are seldom kept over this time period. OK, *A Day In The Life.* But that was <u>one</u> day. And not guitar but a piano!

♣ Approximation for $\Delta W <<« W$

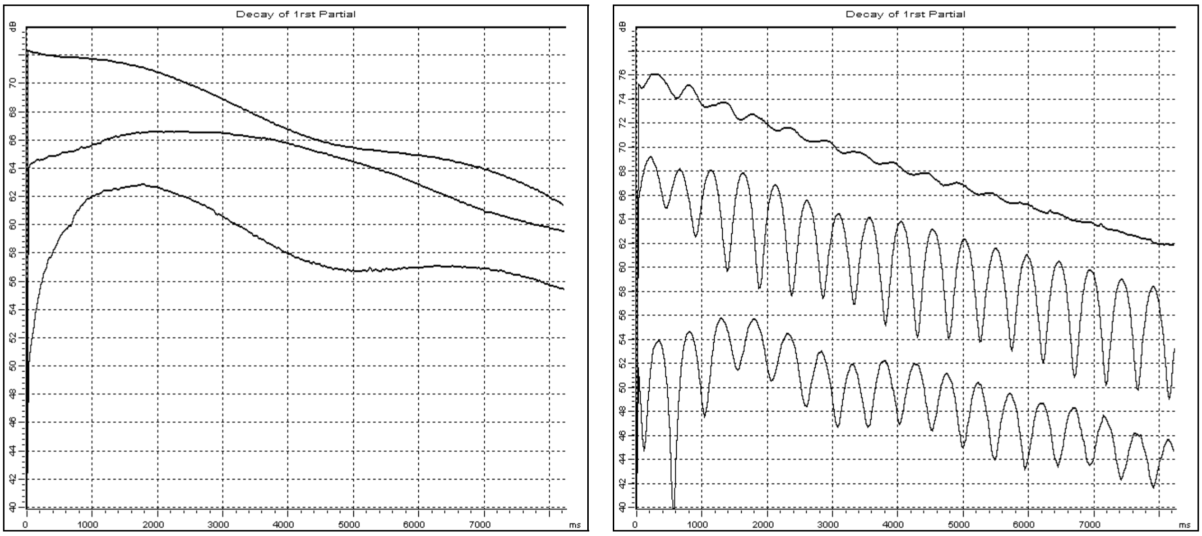


Fig. 4.55: Decay of the first partial of the E_2-string after different excitations; without magnetic field (left), with magnetic field (right).

Since it is discussed over and over again in guitarist circles, if and how the pickup-magnet may dampen the string vibration (and shorten the sustain), we will finally make an attempt at clarification. For this a guitar (Ovation EA-68) was suspended at the strap-button and the E2-string (Fender 3150) was reproducibly hit with a pendulum. An Alnico-5 magnet was attached at a variable distance to the string using a bridge placed over the last fret. The measured signal was generated by the piezo-pickup (**Fig 4.56**). Placing the magnet at 2.5 mm distance causes only a minor level reduction in the 1^{st} harmonic, which hardly stands out. For lower magnet distances the level loss is considerable. At the 2^{nd} harmonic there is an intense beat frequency without the magnetic field; a weak magnetic field (b) will increase the level, a strong magnetic field (c, d) will lead to a substantial level loss. Almost contrary is the 3^{rd} harmonic: here a weak magnetic field (b) will yield intense beat frequencies. The differences in the higher harmonics are so small that they are of the same order as the reproducibility.

From these measurements it can be concluded that the magnetic field *changes* the decay of the partials; the term *dissipation* is conditionally justified only for the first two harmonics – the magnetic field is indeed extracting energy from them in a considerable amount. However, one has to take into account that, in practice, the neck-pickup-magnet is never brought as close as to a distance of 1 mm to the string: the string would otherwise impinge on the magnet. The **small distances** were chosen for the measurements in order to generate a distinct effect. Dissipation effects are only clearly visible for this atypical situation (Fig. 4.56 upper left, curve c and d). During the first seconds the level of the first partial decays much faster than later on. The cause for the higher fluctuation frequency of 4 Hz, as compared to curve b, is the higher negative field stiffness, which will lead to larger detuning. The time-dependent slope of the envelope curve has to be attributed to a non-linear dissipation effect or an amplitude-dependent damping. This is probably due to **hysteresis losses** in the string. As the magnetic field strength in front of the magnetic pole is very inhomogeneous (location-dependent), the field strength and flux density will change within the string during decay. The respective reorientation events within the microstructure are partially irreversible.

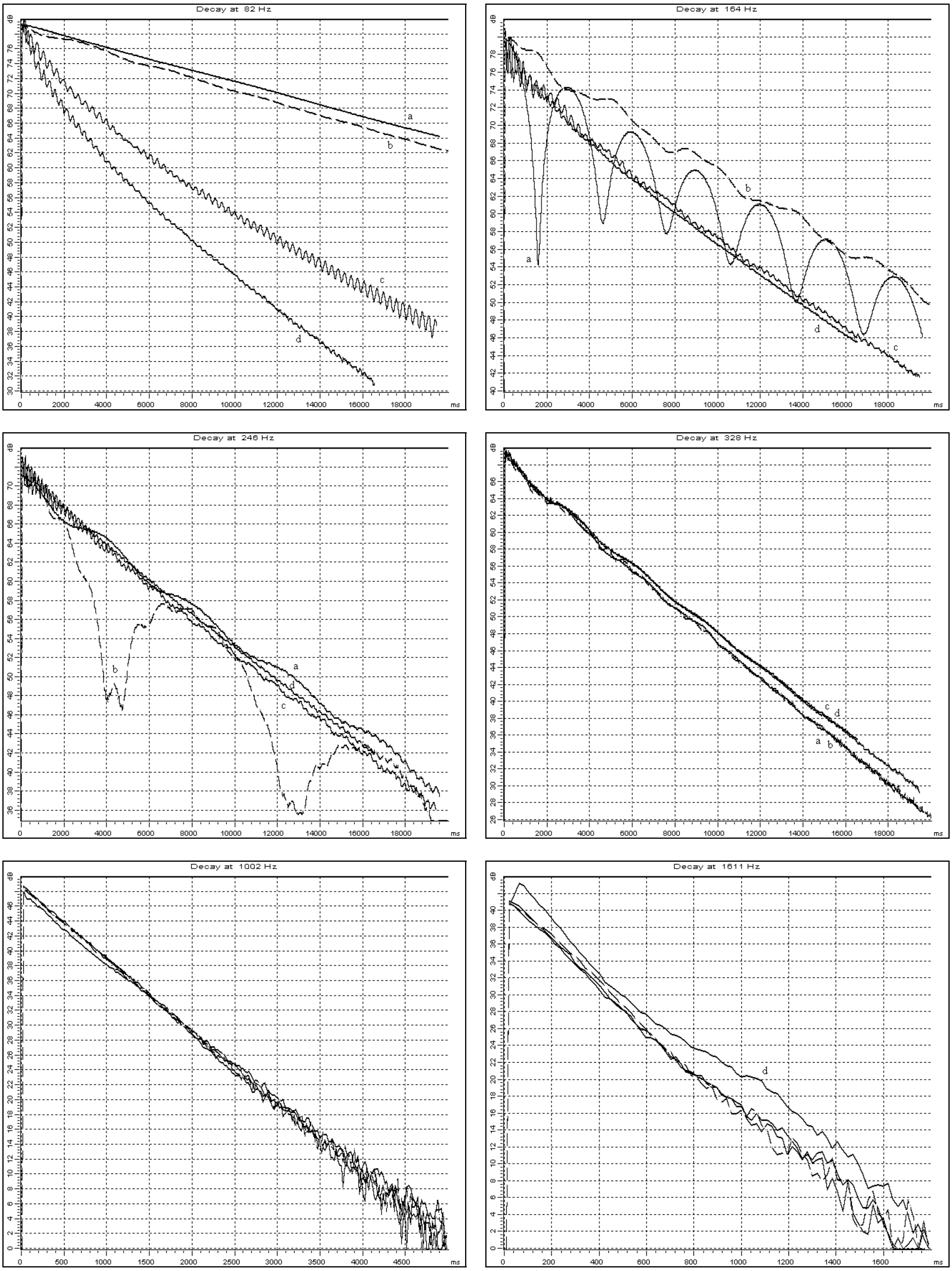
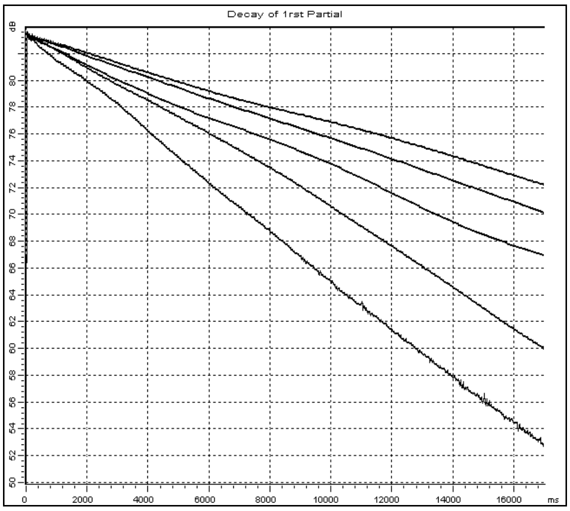


Fig. 4.56: Decay of the partial tone levels of the E_2-string after consistently comparable fretboard-normal excitations: Without a magnetic field (a), a magnet distance of 2.5 mm (b, dashed), a magnet distance of 1 mm (c) and a magnet distance of 0.8 mm (d). The higher d-level trend at 1611 Hz has to be attributed to a slightly different excitation, which shows up only at high frequencies. The results are typical for the guitar under investigation, its specific mounting and excitation; they should not be generalized for other guitars.

The hysteresis losses are proportional to the frequency, in the first approximation. With every cycle of the *BH*-hysteresis loop the magnetic field will lose an amount of energy ΔW; the higher the frequency the higher the number of cycles per second and the higher the dissipation losses. For the string, however, one has to consider that higher frequency partial tones are damped more strongly by other mechanisms and that the strength of the magnetic flux change depends on the displacement. However, the displacement decreases for higher frequencies. The lower pictures in Fig. 4.56 clearly show that the magnetic field does not have any effect in the high frequency range. In addition, for low frequency partials, one should not overestimate the field-induced dissipations. Finally, for comparison, the influence of the **fretting hand** on the decay of the partials is shown (**Fig. 4.57**, left picture). The upper curve shows a measurement in which the guitar was suspended from a steel wire at the strap button, whereas for both of the lower curves the guitar was clamped at the strap button. For the remaining measurements the fret hand surrounded the neck with different tightness but without touching the strings. All measurements were done without magnetic fields. One recognizes that even without magnetic field a variable dissipation is generated – the **heel of the hand** touching the neck has to be interpreted as damping resistance. Its energetic (!) influence on the sustain is considerably larger than that of a common pickup-magnetic-field (right picture).



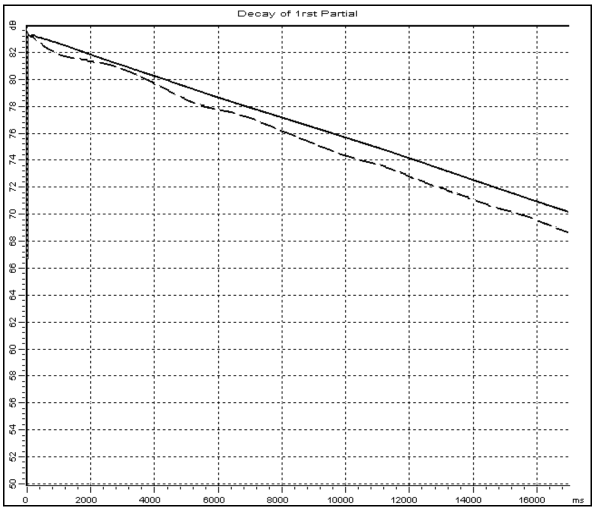


Fig. 4.57: Decay of the first partial tone for different manners of hand-damping (left). On the right, with identical scaling, the influence of an Alnico-5-magnet attached at a distance of 2.5 mm is depicted (neck-position).

4.11.5 Indirect Effects on Sound

In professional music magazines magnet-characteristics are often published without physical rationale. It is to be feared that the following citations are pure speculations resulting from findings after the replacement of an *entire* pickup. In addition, one can only hope that the author also did not replace the strings (… the new pickup delivers much more treble …). For an old Stratocaster pickup, for example, it is impossible to *solely* change the magnets; the coil rests directly on the magnets and as soon as one pulls them out one destroys the flimsy coil-wire. If, however, the whole pickup is replaced by another, the number of turns may change – and, consequently, it would be incorrect to attribute changes in sound only to the magnet.

In literature very different characteristics are attributed to the magnet material, as can be seen by the following collection of citations:

a) “For a pickup with the rather weak Alnico-2 magnet the tone seems to virtually die out after hard plucking. The output signal is not only more quiet but also seems to be less dynamic and perceptibly compressed in the treble range – which is actually appreciated by many guitarists.”
b) “As the magnetic field of an Alnico-II magnet is somewhat weaker than that of a common Strat-pickup (Alnico-V), the string vibration decays more open and more natural. The result is an improvement in sustain.”
c) “Alnico-5: Strong and clear sound.”
d) “Alnico-5: Fast responsiveness and slightly undifferentiated reproduction.”
e) “The stronger the magnet, the more treble.”
f) “As time goes on, older magnets lose some of their power. The less power the magnets have, the better the strings can vibrate. So maybe after 30 years, the magnets are at their 'ideal' power, thus producing 'ideal' tone.” ☺

One might add: “If someone has some Les-Pauls lying around that are older than 30 years – throw them away! Especially for the 50’s Les-Pauls the magnets are completely shot, all power lost, get rid of them. The author will accept these guitars for research sake, at a small waste disposal charge.”

Still, back to the physics. The pickup-magnet is part of a mechanic-electric transducer and as such it influences both the mechanical as well as the electrical partial system. Mechanically the magnet retroacts on the vibrational characteristics of the string; the result can be chorus-like beat frequencies and – to a minor extent – dissipation. The **electrical effect** of the magnet does not really belong in chapter 4.11 because the forces or mechanical effects are described there. The following listing is, therefore, only a precis: The reversible permeability of the magnet influences the inductance of the pickup coil and, consequently, the **resonance frequency**. If the resonance frequency is shifted, partials with different decays may influence the sound and the perceived “sustain”; however, this should not be mixed up with a more openly vibrating string – changes in the cable capacity would have a similar effect. Eddy currents within the magnet influence the **resonance quality factor** (Alnico conducts, ferrite is an insulator). Stronger magnets may increase the **output voltage** of the pickup and overdrive the amplifier in a different way; this may also change the sound and perceived sustain – as well as by changing the input gain. A replacement of the magnets may also change the **aperture** because the spatial flux distribution may change as a function of the (non-linear) string saturation and because the anisotropy of the new magnets may be different from that of the old ones.

The magnetic material can, thus, indeed influence the (“electrical”) sound of the guitar. A hindrance to the free string vibration, however, is not to be expected if the string/magnet-distance is chosen properly.

4.11.6 Coulomb-Force

An electric field with the field strength $E = U/d$ is generated between two electrodes at different potentials. Here U is the potential difference, also depicted as a voltage (or voltage difference, voltage drop) and d is the distance of the electrodes. 100 V at a distance of 10 mm yields E = 10 kV/m. If one inserts an electrical charge q into this electrical field, an electrostatic force F is generated which is called Coulomb-force, after its discoverer (Charles Augustin de Coulomb, 1736 – 1806).

$$\vec{F} = q \cdot \vec{E} \qquad \text{Coulomb-force}$$

The coulomb-force does not play any role for guitar pickups; it may lead, however, to misinterpretations: other than for magnetic forces the coulomb-force also "acts" within the homogenous field. While a positively charged Styrofoam ball between two parallel electrodes is drawn to the cathode (negative electrode) an iron ball between two parallel poles of a permanent magnet will rest (more precise: 4.11.1). Indeed, within the magnetic field there are also attractive forces but they are balanced in this idealized example. The Coulomb-force is only mentioned here to point out its differentness. Analogy-considerations between electric and magnetic fields have model limits that have to be observed.

4.11.7 Lorentz-Force

With the Lorentz-force (Hendrik Antoon Lorentz, 1853 – 1928) we will explain another force that has no direct importance for the magnetic pickup (but indeed for the dynamic loudspeaker). Again, we want to eliminate misinterpretations. A force F acts on a conductor of length l carrying a current I when the conductor is carried into a magnetic field with flux density B. F is oriented normal to the plane defined by I and B. If I is directed parallel to B then $F = 0$. In vector notation one will get the vector product $(\times)$:

$$\vec{F} = l \cdot \vec{I} \times \vec{B} \qquad \text{Lorentz-force}$$

If one points with the thumb of the right hand into the direction of the technical current flow (from plus to minus) and with the forefinger into the direction of the magnetic flux, the middle finger will point into the direction of the force (right-hand rule). The value of the force is given by the product $l \cdot I \cdot B \cdot \sin\alpha$, where α is the angle between the current and field directions. For the magnetic pickup the Lorentz-force, as given in the above form, does not play any role. A small alternating current does in fact flow through the coil which, however, with a value of 10 µA, will not exert any substantial force on it. A retroactive effect on the vibrating string is described by the Maxwell and not by the Lorentz-force, because the string is not carrying a current. If the string would be conductively suspended one could hypothesize an induced current in the neighboring string – however, the effect of its force would be negligible.

4.12 Magnetic Quantities and Units

The literature on magnetic fields refers to two different unit-systems: The MKSA-system as proposed by Giorgi and the CGSA-System.

The **MKSA-system** emanates from the four basic units *Meter, Kilogram, Second* and *Ampere* (SI-units, Système International). All other units are derived from them and occasionally linked with the names of outstanding scientists:

1 N	= 1 Newton = 1 kg m / s^2	1 J	= 1 Joule = 1 N m
1 W	= 1 Watt = 1 N m / s = 1 VA	1 Wb	= 1 Weber = 1 V s
1 T	= 1 Tesla = 1 Wb / m^2	1 V	= 1 Volt = 1 m^2 kg /(A s^3)

The **CGSA-system** uses the four basic units *Centimeter*, *Gram*, *Second* and *Ampere* and derives further units from them:

1 dyn	= 1 g cm / s^2	1 erg	= 1 dyn cm
1 Gb	= 1 Gilbert = 1 Oe cm	1 Oe	= 1 Oerstedt = 1 Gb / cm
1 Mx	= 1 Maxwell = 1 G cm^2	1 G	= 1 Gauß = 1 Mx / cm^2

The following table enables the conversion between both systems:

B	Flux Density Induction	T = V s / m^2	1 G = 10^{-4} T
H	Magn. Field Strength	A / m	1 Oe = 1000 / 4π · A/m = 79.577 A/m
BH	Specific Energy	W s / m^3	1 MGOe = 7.9577 kJ/m^3
Φ	Magn. Flux	Wb = V s	1 Mx = 10^{-8} V s
Θ	Amperes	A	1 Gb = 10 A / 4π = 0.79577 A
F	Force	N = kg m / s^2	1 dyn = 10^{-5} N
P	Power	W = VA = N m / s	1 erg / s = 10^{-7} W
E	Energy	J = N m = W s	1 erg = 10^{-7} J
R_m	Magn. Resistance Reluctance	1 / H = A / (V s)	1 Gb/Mx = 7.9577·10^7 1/H
Λ	Magn. Conductivity Permeance	H = Henry = V s / A Instead of H also Hy for Henry	1 Mx/Gb = 1.2566·10^{-8} H
μ_0	Abs. Permeability of the Vacuum	= 4π·10^{-7} H / m	= 1 G / Oe

4π = 12.566; 10 / 4π = 0.79577.