7.12.2 Non-linear string oscillations

Now, we will have to dive into the thicket of complicated matter. That's because communication engineering teaches us that for non-linear systems there is neither superposition nor proportionality, and neither transfer function nor step response. Of course, we may drive also a non-linear system with a step excitation, but the response (reaction) is not a signal-independent system function, but it depends on the excitation signal – and as a result there is not "the one" step response but there is an infinite number of such step responses.

Strictly speaking, every technical system is non-linear, but often this is to such a small degree that the transmission characteristics may be simplified towards linearity. Chapter 5.8 had dealt with the non-linearities occurring in pickups – the effects are far from insignificant yet they are far outweighed by the non-linearities possible in the string vibrations. The latter become non-linear if, after being plucked, the string hits the frets. In this case, the step-waves generated by the plucking are not only reflected by the bridge and the nut (or the fretted fret) but also at the (other) frets. This process is dependent on the plucking-strength and therefore it is non-linear.

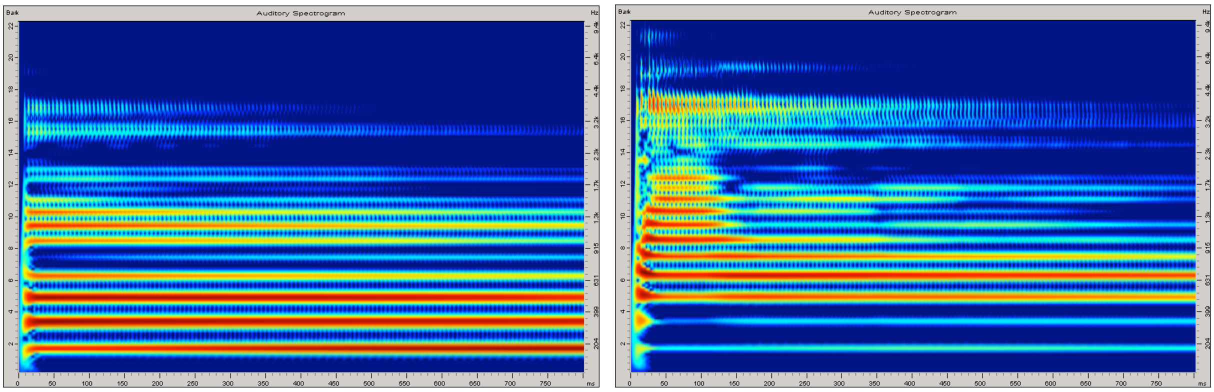

Fig. 7.137: Spectrogram of a plucked D-string (E3, 0 – 800 ms, 0 – 10 kHz, dynamic in the graph = 30dB). Left = lightly plucked, right = strongly plucked. Fender Telecaster, strings 009 - 046, bridge pickup. The analyses were scaled to the same maximum drive level.

Fig. 7.137 shows spectrograms of a D-string plucked with different strengths. Depicted are **auditory spectrograms** (Cortex VIPER) the analysis-parameters of which are adapted to the characteristics of our hearing system. It is hard to believe that both analyses were obtained with the same guitar, the same string and identical plucking positions – only the plucking strength varied. The lightly plucked string clearly reveals the interference filter, with the spectral emphasis being formed by the first three partials. The outcome for the strongly plucked string is very different: the first two partials (fundamental and second harmonic) have only a weak level – their vibrations cannot unfold due to the amplitude limitation. Even if a simple model would attribute the same displacement-amplitude to each partial, the pickup voltage – corresponding to the velocity – would increase with increasing order of the partial in this model. In a real plucked string, the partials do not have the same displacement amplitude: the plucking-interference-filter causes gaps (e.g. for the 5th and the 10th partial). However, already the first string/fret-contact starts to fill in these gaps. If we interpret the plucking of the string as a **step-excitation**, the string hitting the fret could be seen as a kind of **impulse-excitation**, albeit quite a special one. This is because while the plucking action feeds vibration energy to the string, hitting the frets can only cause an energy loss. How big this loss is depends on the surface qualities (among other factors): little loss for a fresh string and a clean fret but more loss, if an in-between layer of dust/grease/talc acts as an absorber.

The (normalized) time function shown in **Fig. 7.138** underlines the differences between the lightly and the strongly plucked string: during the first few periods, only the dispersion has an impulse-changing effect in the left-hand graph, while for the strongly plucked string, reflections are clearly visible already within the first period. These are reflections that can only stem from the obstacle located closest – and that is the last (22^{nd}) fret. There is a significant chance that a strongly plucked (thin) string comes in contact with the last fret already in the plucking process (compare to Chapter 1.5.3), but the exact evolution over time of this and other fret-contacts are dependent on the individual plucking – this is in fact why the system behavior is non-linear. A model can therefore emulate the fret-bounce either only for the individual case, or simulate – as a stochastic model – a generic average event. Which is the problem: we will not get far with one model alone, because the well-versed guitarist is able to generate a *multitude* of fret-bouncing "snap-sounds".

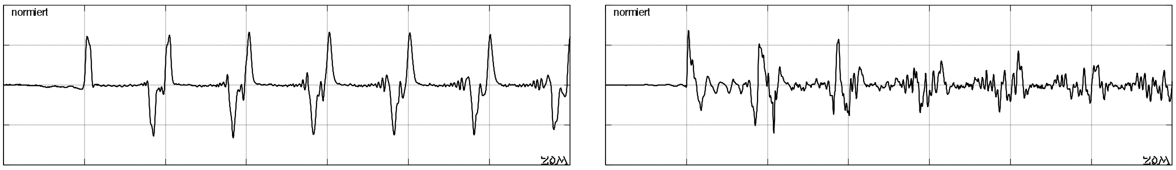

Fig. 7.138: Time function of the pickup voltage; left = lightly plucked string, right = strongly plucked string.

Fig. 7.139 depicts in which unexpected variants the decay of a strongly plucked string can occur. Again, the D-string of a Telecaster fretted at the 2^{nd} fret is shown, strongly plucked at 12 cm distance from the bridge (as in Fig. 7.137). As opposed to the above analysis, the string was not pushed downward at a slant, but lifted up and then let go. At 0.6 s, the spectrum (and the sound) change unexpectedly: the 5^{th} partial literally cuts out, while other partials only come to life at that point in time. These changes are not connected to the fretting hand but are the work of the string alone – in cooperation with the frets.

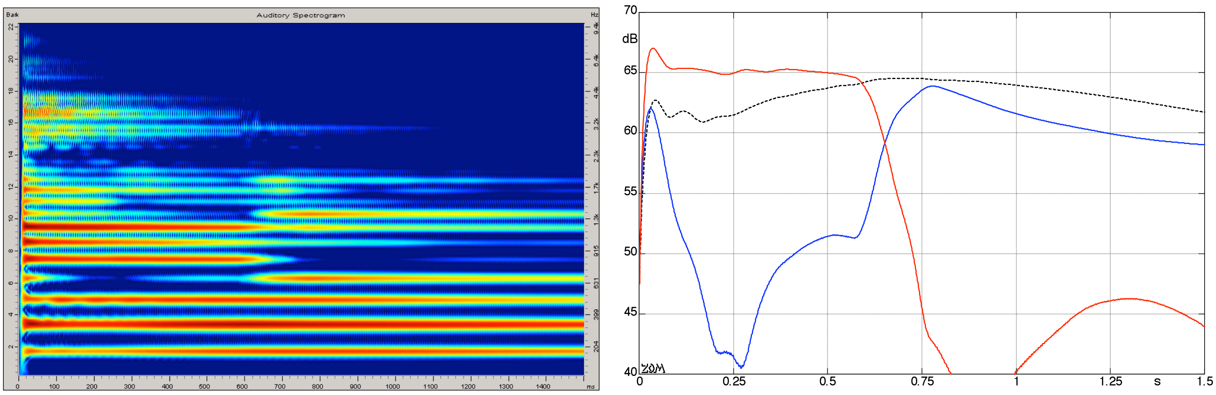

Fig. 7.139: Spectrogram of a plucked D-string (E3, 0 – 1500 ms, 0 – 10 kHz, dynamic = 30dB).
Right: red = 5^{th} partial, blue = 4^{th} partial, black = level of fundamental; all as a function of time.

The string, with a vibration that is at first almost perpendicular to the fret-board, hits the frets which transfers part of the vibration energy into a mode parallel to the fret-board. Over time, however, the plane of vibration changes back again, as easily visible from the level of the fundamental (----). Around 0.6 s, the increasingly fret-board-normal vibrating string approaches the frets again such that a further crash occurs. This crash considerably disrupts the 5^{th}-order vibration, but at the same time retriggers and amplifies the 4^{th}-order vibration. A model describing vibrations in only a *single* plane would not succeed for such a behavior, even if that model would allow for non-linear amplitude limiting.

Fig. 7.140 shows further spectrograms; again only the strength of the plucking was varied. It is characteristic that the partials decay neither exponentially nor according to a simple beating-model, but suddenly change their decay behavior. Even an increase in level is possible (albeit one only for a limited time) – this can be attributed to a slowly rotating polarization plane. Contact between string and fret may be limited to the first 0.1 s, but may also still occur after 1 s. The evolution of the level and the sound color over time is correspondingly rich in variation. These figures highlight that the neck (or rather the frets) enjoy an elementary significance: a fret minimally projecting over the other frets will generate other bounce-contacts than one that is worn down.

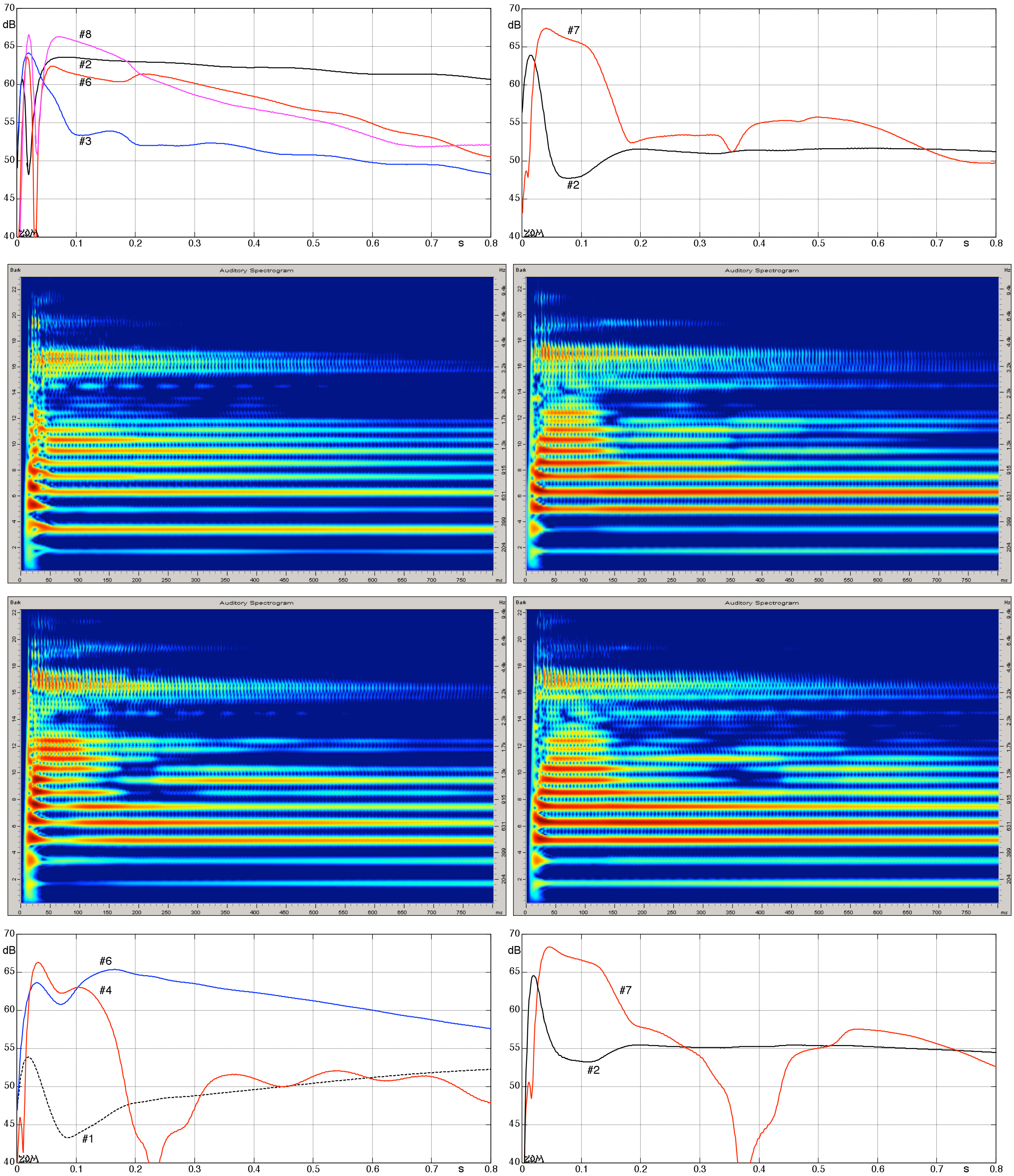

Fig. 7.140: Auditory spectrograms and levels of partials of a plucked D-string (compare to Fig 7.139). Level-normalized scaling; dynamic in the graph = 30 dB. Telecaster, bridge pickup, fresh strings (009 – 046).

It is not difficult to corroborate speculations about string/fret-contacts by measurements: for this, all 22 frets of a **Telecaster** were electrically connected to a 22-channel analyzer, and all contacts occurring during the decay were stored. The representation in a **tactigram** (bounce chart) shows characteristic patterns that agree well with a line model (**Fig. 7.141**). The D-string of the Telecaster is pushed down such that it comes into contact with the last (22nd) fret – this happens often when light strings are used. As the string looses the contact to the pick, waves propagate in both directions and are reflected at the last fret and at the bridge, and lift the string off the fretboard, as shown in **Fig. 7.142**.

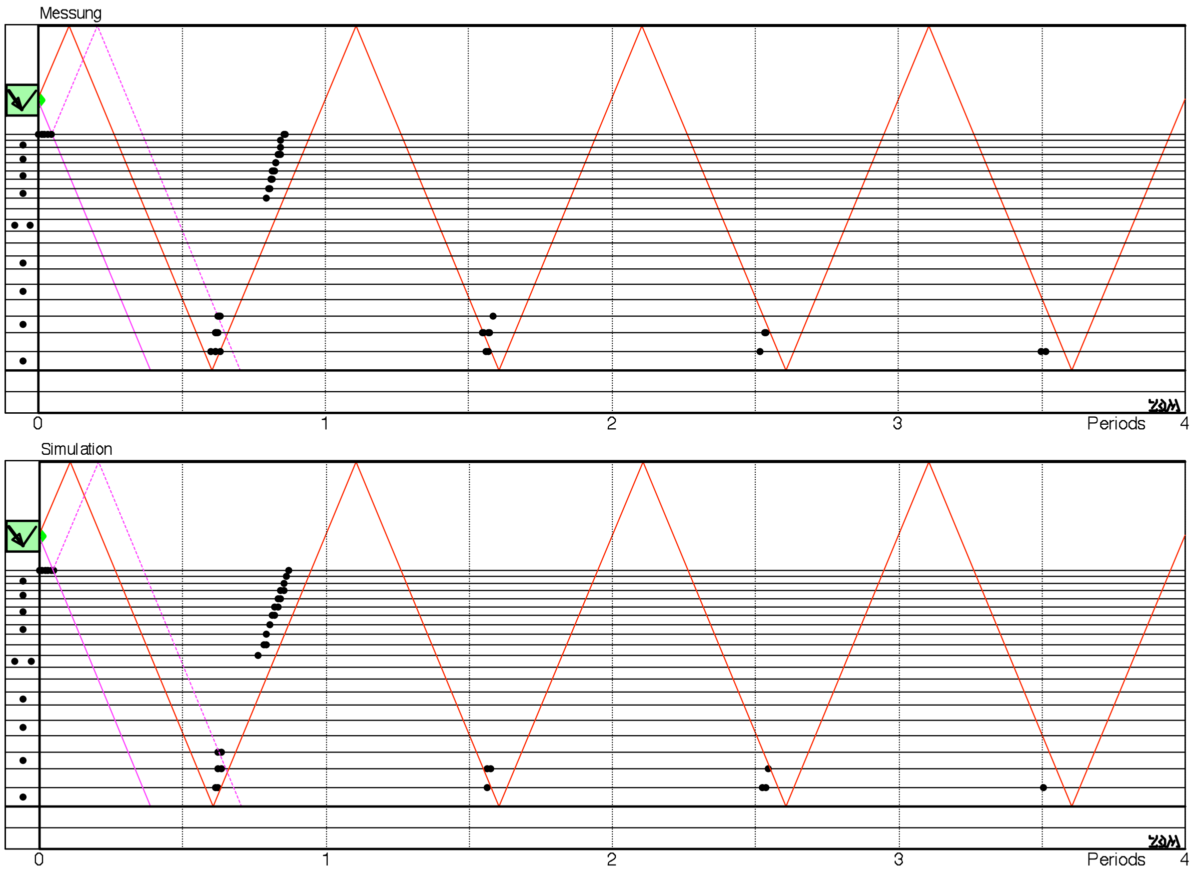

Fig. 7.141: Bounce chart. Telecaster, D-string fretted at the 2nd fret, pressed down strongly 12 cm away from the bridge and then released. Top: measurement; bottom; model-calculation. Dots = string/fret contacts.

After half a vibration period, a maximum in the displacement has formed above the 6th fret (**Graph #8** in Fig. 7.142); it breaks down again during the further continuation of the vibration. Immediately afterwards, the string hits the fretboard, with curvature of the neck and condition of the frets deciding where exactly the string/fret contact happens. The angle with which the string is pressed down also plays a role: it makes for a difference whether the string is pushed down exactly perpendicular to the fretboard or with a slant relative to the fretboard. This is because the orientation of the string excitation determines the share of the fretboard-parallel vibration. During the decay process, the plane of vibration rotates (even specifically to each partial), and it is in particular the fretboard-normal share of the vibration that is clipped by bounce-processes. The fretboard-parallel vibration-mode is a kind of energy-storage that only slowly feeds its vibration energy to the fretboard-normal vibration. The latter (being important for the pickup signal) can therefore repeatedly generate further string/fret contacts. Note that in the model calculation shown in Fig. 7.142 only one plane of vibration was considered.

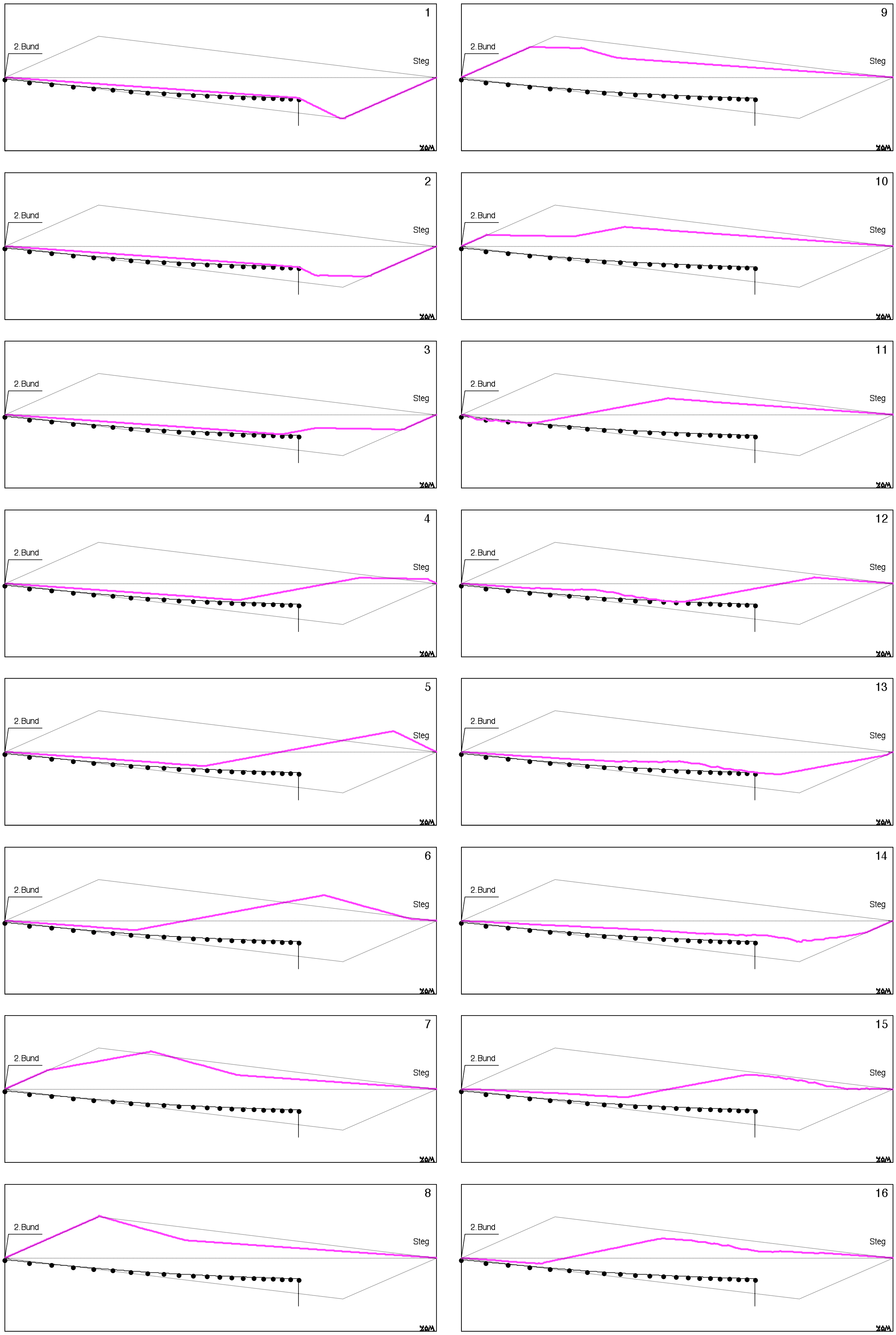

Fig. 7.142: String displacement at various points in time. Parameters as in Fig. 7.141.

Fig. 7.143 shows how big the spectral differences between strongly and lightly plucked strings can be: in the left column we see, during the first milliseconds, the **attack-spectrum** already known from Fig. 7.136 (red envelope, interference gaps dependent on the pickup position); it transitions into the **decay spectrum** (blue envelope). In the column on the right, a sinc-shaped envelope cannot form at the beginning due to the string/fret contact supplying additional impulses. Only later, an influence of the pickup position establishes itself as an outline, while the plucking-position is not evident anymore at all as interference filter.

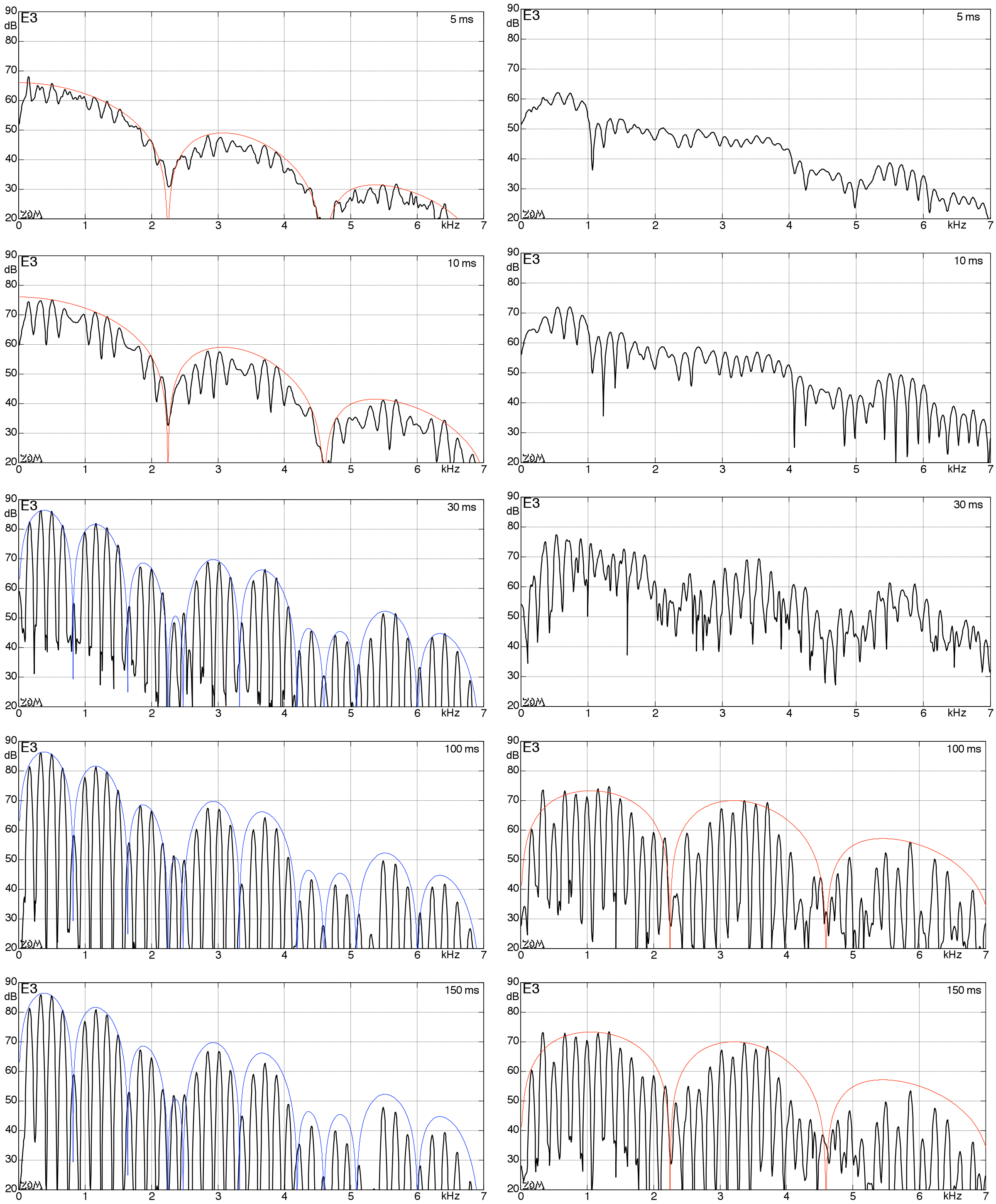

Fig. 7.143: Spectra of the pickup voltage (subjected to a window). E3 on D-string, Telecaster (cf. Fig. 7.136). Left = lightly plucked string, right = strongly plucked string. Level-normalized representation.

An entirely different vibration happens if the string is not plucked pressing down, but is lifted and then released. This plucking technique is found especially with guitarists playing finger-style (i.e. without a pick) – but even with a pick, an at least similar behavior can be achieved. **Fig. 7.145** shows snapshots for a string pulled up far enough so that it bounces on the frets after its release. In this example, first contact happens at the last (highest) fret, followed by a series of contacts running along the fretboard towards the lower frets. Then (**Graph #7**), the string looses contact only to touch all frets in quick succession (**Graphs #13 – 14**). Or at least almost all frets – in details this of course again depends on minute differences in the heights of the frets.

Fig. 7.146 once more compares contact-measurements with model calculations. Considering the complexity of the matter, the correspondence is very good at the start – as they progress along the time-axis, the two representations differ more considerably. This is because dispersion was not modeled, because the polarization was only calculated for one plane (and not circularly), and because the fret-heights were idealized in the model (in the investigated Telecaster specimen, the fret were already slightly worn).

Fig. 7.147 indicates that string/fret contact is not necessarily limited to the attack-phase. In this example, the string repeatedly bounces off the 3rd fret – however this happens so lightly that no annoying buzz but merely slight brightening of the sound (a mixing-in of treble) occurs.

We see from **Fig. 7.144** how strongly even tiny differences in the height of the frets can make themselves heard. Here, we first calculated the string velocity over the pickups using the non-linear string model, and then derived the spectrum from it. This was done for two different fret-boards on which the 18th fret differed in height by 0.2 mm.

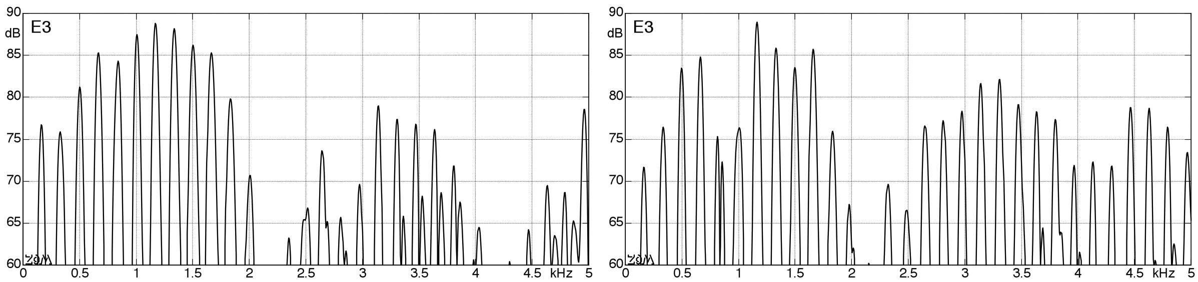

Fig. 7.144: Calculated spectra of the D-string bouncing off the frets. The only difference between the two graphs is that the height of the 18th fret differs by 0.2 mm.

These results give an indication of what can happen when comparison tests are run by a magazine checking out the "holy grail" – i.e. if, for example, a original 1950's Les Paul is compared to a more recent reproduction. Of course, the frets of the priceless♣ vintage guitar are worn, maybe so strongly that it causes the celebrating tester to grimace a lot, and of course the trained ear will hear all kinds of differences. Too bad: as soon as this "grail" is put in a playable condition, its $-value takes a nosedive. Thus do note: on every grail rests a curse of some kind.

♣ Not to be taken all that literally: that's from about € 200.000; quite nicely done fakes may be acquired.

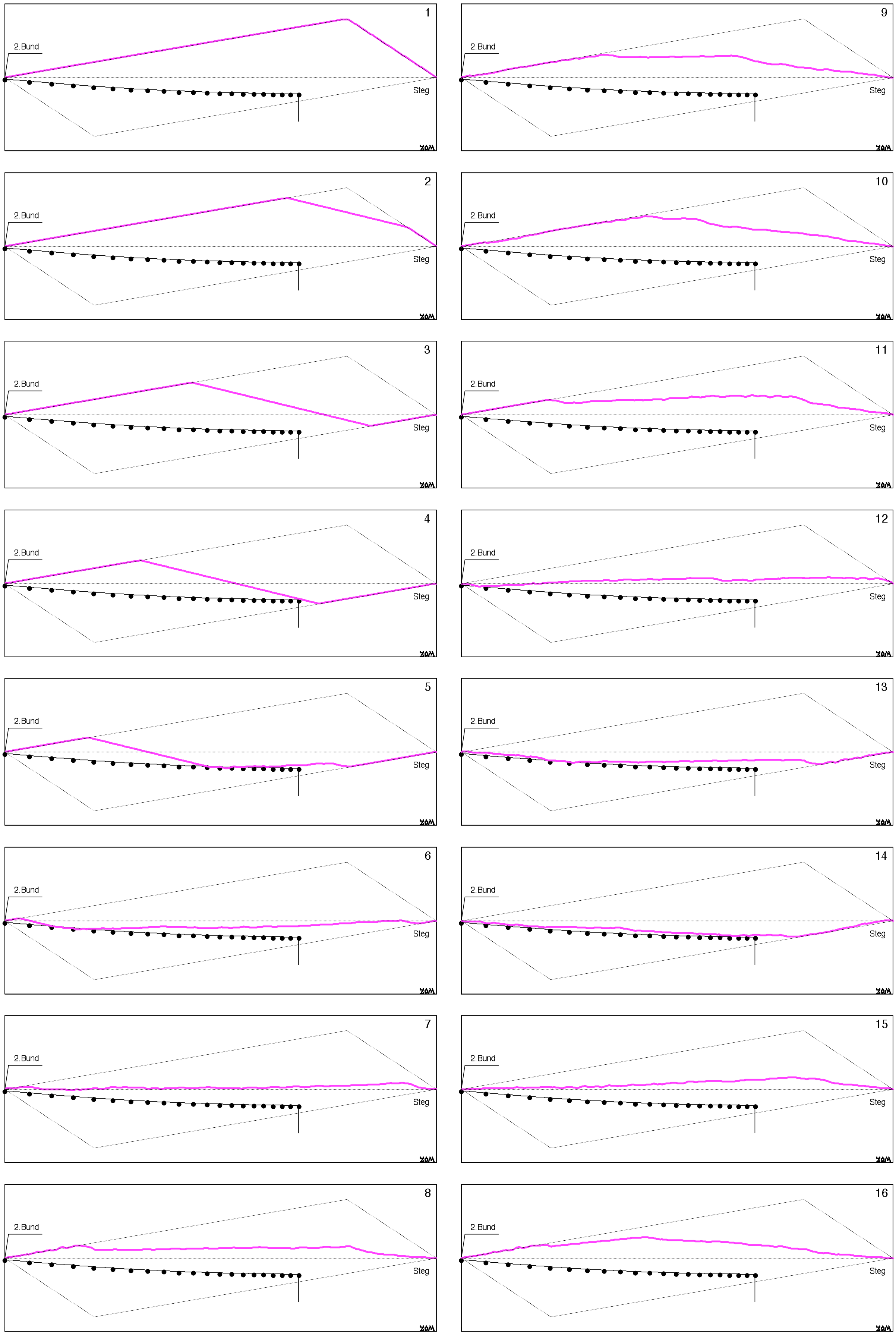

Fig. 7.145: String displacement at various points in time. Parameters as in Fig. 7.141.

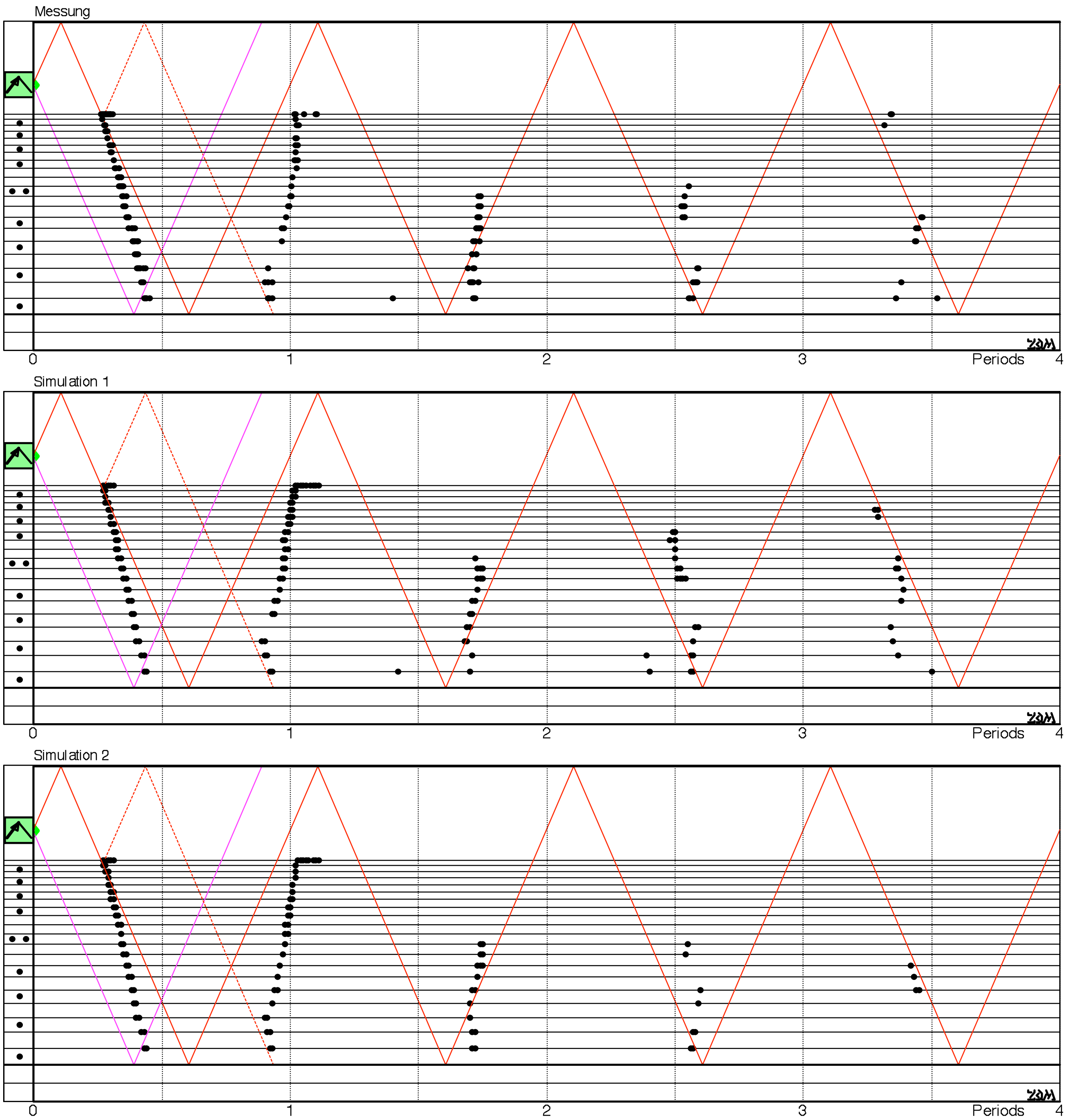

Fig. 7.146: Bounce chart. Telecaster, D-string fretted at the 2nd fret, pulled up at 12 cm from the bridge and released. Top: measurement, middle and bottom: model calculations. Dots = string/fret contacts.
For simulation 2 (bottom), the bridge was raised by 0,3 mm relative to the setting for simulation 1.

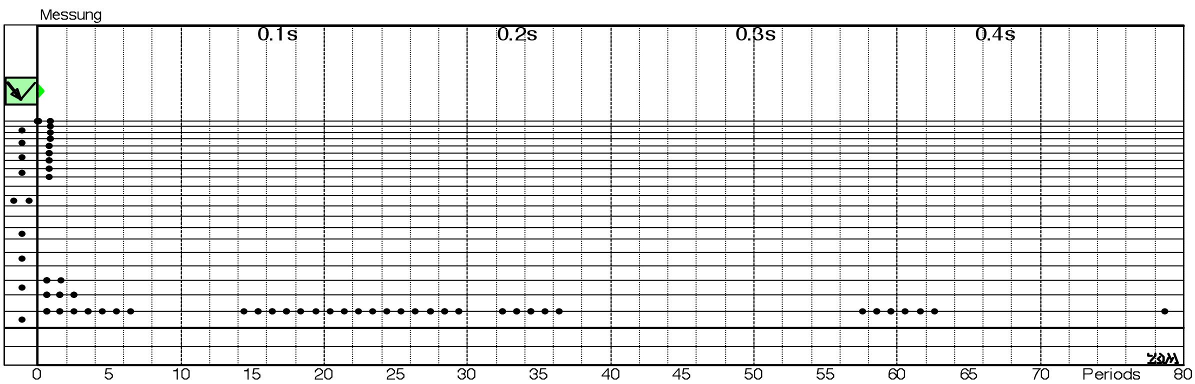

Fig. 7.147: D-string fretted at the 2nd fret: even after 0.5 s there are string/fret-contacts

In a nutshell, we have the following situation (for further elaborations see Chapter 7.12.3): In electric guitars with heavy strings and a high action, string/fret contacts (other than the actual fretting action) are rather rare, the low partials can develop nicely, and the electric sound is quite full. The string vibration can approximately be modeled in a linear fashion. Given light strings and the correspondingly connected light playing forces (Chapter 7.4.1) each individual note may be accompanied by string/fret contacts (especially for strong plucking), resulting in a more percussive sound with more treble. The low-frequent partials are less distinct because they lack the required amplitude. Of course, the individual plucking process always is essential: with brute force, it is possible to make a heavy string bounce onto the frets, as well, and a light string may be plucked so gently that it does not come into contact with the frets. That's what this statement is based on: **it's all in the fingers, man!**

For short notes, the **guitar body** has next to no influence on the electric sound, and for solid body guitars no influence is felt for longer sustained notes, either. With hollow-body instruments, in particular two effects are found: since especially the low-frequency notes are (acoustically) radiated better, the corresponding decay times are shorter, and for the same reason these instrument tend to feed back more quickly.

In terms of influencing the sound, the way/style of playing comes first, and strings and pickups are next (in high quality guitars). We then get to the mechanical characteristics of the bridge, and then to the frets (even the higher-most, possibly "never used" ones). That the acoustical sound radiated by an electric guitar would give "complete" testimony about the electric sound is a fairytale – albeit one that apparently cannot be silenced. Already Leo Fender and Les Paul fully understood that the vibration-energy needs to remain in the string as long as at all possible – as little as possible should be transferred into the body. Any acoustic sound needs to be channeled through the body (to use layman's terms) – so the material it is made of is relevant, but – alas! – only for the acoustic sound. The guitar body can influence the electric sound, but only in terms of absorption. Since it seems that every guitar player demands a sustain as long as possible, the absorption needs to be as low as possible. In that case, however, the influence of the body wood on the electric sound has to be as small as possible, too. Knowing that, it is not surprising that an electric guitar build from undefined, knotty platform-wood can fill the guitar player with enthusiasm due of its sound (G&B 7/10) … because of its electric sound, that is, of course.

7.12.3 The roots of the electric sound

Of course, the pickup voltage does not yet yield a "sound" – for that, amp and speaker are required, and – diving into philosophy – a listener, as well. Wouldn't it make for a great debate to ask whether airborne vibrations that are not heard by anybody merit the term "sound"? But that would be the realm of those physicists who – good heavens! – seek to become a DPhil rather than a DSc, i.e. move into a world completely foreign to the Doctor of Engineering. In short: without amplification, the electric guitar generates an acoustic sound, amplified it generates the electric sound. Only the latter is addressed in the following, as is the analysis and description of its origin.

Step-excitation and pick-filter

From a systems-theory point-of-view, plucking a string represents an impressed force-step – however not one in the form of an ideal step-function but modified by the pick-filter (Chapter 1.5.2). Due to mode-coupling in the bearings (bridge, frets) and magnetic pull-forces, the string vibration does not remain in one plane but starts a wobbling motion in space (circular